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ORIGINAL RESEARCH

Competitive Environment Between Green and Non-green Products Considering Disruption and Alliance Strategy

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Abstract *Nowadays, the effective supply chain is considered more than ever. In this study, a supply chain composed of a dominant retailer and a non-green product supplier is investigated. Base demand is always fluctuating in the real market. Thus, considering the base demand disruption risk is one of the factors approaching the problem to reality as discussed in this model. Then, the retailer encounters the challenge of entering a new supplier producing the substitutable product of the first product. New supplier produces the green product. Thus, the competition of green and non-green products in this study is considered with other factors. Another challenge for retailer is the type of optimal alliance, determining the optimal green degree of the second product, and determining the optimal refund amounts. In this study, some factors such as base demand disruption risk, green product, alliance selection, and return policy are considered. A game theory approach is used for solving the problem and getting optimal decisions. Finally, some sensitivity analysis based on the alliance strategies and greenness of the SCM has been done by numerical examples.*

Keywords Alliance strategy · Disruption management · Game theory · Green product · Pricing · Supply chain

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Introduction

Today, customers in real market are ever-increasing powerful; therefore, an individual's customer satisfaction drives the necessity for having an innovative and effective supply chain. Generally, problems with the coordination between supply chain members are highlighted because of the increasing competition among supply chain members. Researchers are engaged on different policies to achieve this coordination (Leng 2005; Kumar et al. 2013; Singh et al. 2019). As a result, different types of policies and strategies are implemented for enhancing the effectiveness of the supply chain. Such strategies are return policies (Zhu 2012; Hu et al. 2014), revenue sharing policies (van der Rhee et al. 2014; Zhang et al. 2019), quantity discounts (Zhou 2007; Tsai 2007; Peng et al. 2018), etc. One new method of coordination in supply chain is strategic alliance which has been recently considered by researchers (Amrouche and Yan 2013; Karray and Sigué 2015). In most of the researches, supplier has the power within the supply chain (Zhou et al. 2008; Liu et al. 2019). On the other hand, supplier is not the foremost powerful member of the supply chain all the time. Retailers can play an active role as a robust member within the market. Thus, retailer can be the dominator in the supply chain (Zhou et al. 2015; Taleizadeh et al. 2017). This study investigates the pricing issue with strategic alliance in a supply chain which the retailer is a leader and new potential supplier wants to enter the specified supply chain. In reality, we face with dynamic and volatile market, and our supply chain would face with the arrival of new members. Consequently, the concept of new member is essential to be implemented within the model.

Amrouche and Yan (2013) stated that there is a highly significant difference between strategic alliance among the supply chain members and vertical integration. Strategic alliance implies that supply chain members share all their power to achieve the common goal but work independently. Vertical integration means that two different stages of supply chain (e.g. producer and retailer) work under the identical management to extend their power in the market (Amrouche and Yan 2013). In this study, different strategies of alliance among the supply chain members are studied. Because of the rise of competition among the retailers in today's market as well as the variety of products, the supply chain members must seek for encouraging policies for customers to possess more sale and increase the number of customers in supply chain. It is argued that customer loyalty is an essential factor for supply chain success which is influenced by retail price and retail services (Yuen and Chan 2010). Thus, in today's competitive market, customers have more power in the supply chain. For this reason, different policies must be implemented for satisfying the customer. As mentioned, return policies are one of these policies. However, nowadays the product return is not only due to the adverse quality, but other reasons such as the difficulty of product installation and implementation, incompatible performance of product, customer performance, or customer regret can cause the product return (Shulman et al. 2011). Furthermore, the optimal refund amount must be determined because although the full refund amount increases customer satisfaction, it is not in favor of the producer (Xu et al. 2015). Therefore, determining the optimal refund amount is one of the factors discussed in this study.

Another factor which can bring competitive advantage to the modern supply chain is green products (Basiri and Heydari 2017; Marić and Opazo-Basáez 2019). The issue of sustainability in supply chain has been emphasizing due to the social, environmental and economic aspects (Hák et al. 2016). The population is increasing according to the reports (UN 2017). Thus, producers have to consider the customers' needs and environment simultaneously. In this regard, producing products with appropriate quality and lowest negative effect on the environment is concerned in researches (Hong et al. 2019). Environmental aspect of sustainability is essential to be modeled in the researches (Everard and Longhurst 2018). In addition, customer's awareness of climate change and pollution makes manufacturers to produce environment-friendly products. In the industry with increased production, the use of fossil fuels and pollution is increasing (Mahmoudi and Rasti-Barzoki 2018). Hence, achieving the goal of sustainability through producing green products has been discussing (Madani and Rasti-Barzoki 2017). Such concerns guide the manufacturers toward producing green products to contend with harmful effects of the supply chain activities (Jakhar 2014; Shibin et al. 2016). As a result, studying the green product, its effect on supply chain, and green degree of products are among the factors discussed in this study.

When green products are produced, customers have to choose based on their priorities. As a result, the demand between choosing the green and non-green product is considered. Due to the variety of the products, it is crucial for firms to produce based on the real demand. Thus, it is essential for firms to plan their approach by considering the disruptions (Waller and Fawcett 2013; Schniederjans et al. 2020). In general, most studies on supply chain management are conducted under normal conditions. In other words, the market demand is constant and the producer has the full information. In real conditions, the human and natural factors can cause demand fluctuations in the market. Disruption risk management minimizes the negative effects of risk on the performance of supply chain (Ali et al. 2018). Demand is one of the factors that is very effective in supply chain performance. Demand disruption occurs due to the factors such as losing the main customers, creative competitors, and inappropriate prediction of demand (Koblen and Škurková 2015). Consequently, the demand disruption is considered as probable in the model and its effect is studied numerically in the two-stage supply chain where the retailer is leader and the suppliers are followers.

Consequently, as mentioned above, we studied different types of alliance strategies in a two-echelon sustainable supply chain composed of retailer and supplier in the presence of disruption risk and we investigate the effect of some parameters on the green degree in four alliance strategies. We determine the optimal value of refund amount, retail price, wholesale price, favorable margin of revenue and green degree in all possible alliance strategies. Also, we provide insights into the effects of green degree in all possible alliance strategies and the impact of the arrival of a new member into the supply chain. We determine some factors which can change the optimal alliance strategy. In all previous statements, we consider return policy and its effects on our model. To best of our knowledge this is the first time these factors are modeled together for improving our decisions in the real market. Following questions are answered in this study:

1. Is it beneficial for the retailer to accept the arrival of a green product?
2. Which parameters can change the optimal alliance strategy?
3. What is the effect of return rate in the supply chain in all kinds of alliance strategies?
4. How probability of demand disruption can change the green degree in all kinds of alliance strategies?

5. How the retailer can coordinate the supply chain by considering the demand disruption between green and non-green products?

The sections of this paper are organized as follows. "Review of literature" section provides a review of literature. "Problem Statement" section presents the structure of the studied supply chain and defines the four alliance strategies. In "Model Formulation" section, we formulize the different models. In "Optimal Solutions" section, optimal solutions for different strategies are discussed. A procedure is proposed in "Decision-Making Stages of Optimal Alliance Strategy Selection" section for selecting the optimal alliance strategy. Numerical analysis is presented in "Numerical Example" section. In "Sensitivity Analysis" section, the sensitivity analysis is discussed. Finally, in "Conclusion" section the summary of our research is provided.

Review of Literature

The form and selection of alliance type in supply chain is a significant issue. In this study, the word "alliance" means that the various supply chain members cooperate and have benefit sharing mechanism to own more sale and profit (Taleizadeh et al. 2017). There is little research on alliance selection in supply chain system. Gayle and Brown (2014) presented a model about airline indicating how alliance can affect demand and supply. In addition, Amrouche and Yan (2013) studied the effect of strategic alliance on decision-making associated with to pricing in a supply chain composed of a producer and two retailers. They found that strategic alliance cannot be always profitable for supply chain members. Karray and Sigué (2015) investigated the supply chain composed of three manufacturers that in this chain, two manufacturers of complementary product and one manufacturer of independent product worked. In their study, three scenarios are studied by using the game theory such as no promotional partnership is among the members, promotional partnership is between the producers of complementary products, and promotional partnership is between all three members. Thus, the optimal case of alliance and partnership is selected after reviewing three scenarios. Nguyen (2019) studied performance evaluation in a construction industry and used up-to-date data envelopment analysis method to extend the effectiveness of the choice. The issue of finding the optimal alliance strategy was discussed to provide managers beneficial insights. Zhou et al. (2015) studied the pricing issue with alliance selection in a supply chain where retailer plays a role as leader and the upstream member can enter the supply chain.

Market is not constant in the economic system and real world. Thus, a new member can enter the supply chain at any time. Hauser and Wernerfelt (1998) studied the decisions on competitive pricing when a new member enters the market. Little research was conducted on the challenges related to the arrival of new members. So that, many researchers such as Tyagi (1999) and Schultz (1999) considered the arrival of a downstream member such as retailer to the chain and studied its effect on pricing. In addition, some studies were conducted within which the supplier prevented the arrival of a new supplier to the supply chain by considering some limitations such as the study of Xiao and Qi (2010). Then, Arbatskaya (2001) considered a supply chain using low-price guarantee strategy for preventing the arrival of a new member to supply chain. As it is mentioned above, Zhou et al. (2015) studied the arrival of an upstream member by implementing alliance selection for coordination. In their research, return policy is not considered as a promotional policy.

Todays, many promotional policies are considered in addition to the pricing policies which are used for customer attraction due to the intense competition between the retailers selling their similar products via traditional and online channels. For instance, return policy is one among these policies. In return policies, the refund amount in purchase of consumer and decisions related to return are very significant. The effect of partial refund amount and full refund amount on customer return was studied by Mukhopadhyay and Setoputro (2004). In the mentioned study, the effect of return policies and pricing strategy on consumer purchase and product return decisions in an online sale channel is studied that the return amount in that demand only depends on return policy. Ai et al. (2012) studied the case where two supply chains producing substitutable products compete with one another. In the two desired supply chains, there is demand uncertainty and pricing issue is studied in two states of presence and absence of full return policy indicating that the role of full return policy when there are two competitive chains differs from when there is just one supply chain. Xu et al. (2015) investigated the return policies when the customer evaluation depends on the refund amount and the time when the customer can return that product. Based on the study conducted by Su (2009), full refund causes adverse and unnecessary returns. Javadi et al. (2019) defined a model which considered environmental concerns. This study investigates optimal pricing under governmental policies in a dual-channel supply chain and different types of return policies. Li et al. (2019) investigated a supply chain with a manufacturer which can sell its product in online and traditional channels. In their study, they search about the effects of four return policies: full refund amount in the online channel, full refund amount in the traditional channel, full refund amount in both channels, no refund amount in both channels. As a result, the optimal refund amount must be considered too. Thus, Taleizadeh et al. (2017) studied pricing and alliance selection by considering the return policy and arrival of a new member to the supply chain. In their study, they ignore the effect of risk in the model.

Supply chain risk management is a systematic approach for identifying, evaluating, reducing, and controlling the disruption risks in supply chain which are considered for controlling the negative effects of risks on supply chain performance (Aqlan and Lam 2016; Aldrighetti et al. 2019). In line with the effective management of supply chain, organizations analyze and identify the internal and external risks. Chen and Xiao (2009)studied a supply chain composed of a producer, dominant retailer, and several marginal retailers. In their study, the coordination between supply chain members is studied when there is the demand disruption risk. Huang et al. (2012) studied the production and pricing issue for a two-period and two-channel supply chain composed of a producer and a retailer when there is a demand disruption risk. Cao et al. (2013) investigated the coordination mechanism in a supply chain composed of a manufacturer and n retailers when the demand and cost disruptions are considered simultaneously. In addition, in their study, the coordination mechanism is used by considering the revenue sharing and the effect of these two risks on the contracts of revenue sharing to select the optimal strategy for supply chain members. Aqlan and Lam (2016) investigated the supply chain optimization and simulation techniques for risk management in supply chain. Pi et al. (2019) considered pricing and service strategies in a supply chain which consists of a manufacturer and two retailers. In the mentioned supply chain, manufacturer sells its product through online channel and two retailers in the presence of demand disruption. Paul et al. (2017) studied a three-stage supply chain composed of several manufacturers, distributors, and retailers. Three approaches are presented in their study: the first approach presents an ideal program for unlimited time horizon and a program for the time any change occurs in data. The second approach presents a program for managing predictive demand changes. Finally, an approach is presented for studying the sudden disruption management of production. Ali et al. (2018) investigated the price and service levels in a supply chain composed of a producer and several retailers with demand disruption risk. The mentioned study deals with supply chain in centralized and decentralized states. Based on previous analyses, the prices and service levels are affected by demand disruption risk. Evaluating risks in green supply chain is considered in Mangla et al. (2015). Rahmani and Yavari (2019)defined a model which demand disruption risk is considered in a dual-channel green supply chain. In their model pricing, greenness and production variables are formulated. Because of the importance of having environment-friendly products, researchers have started considering the concept of green supply chain and its effects.

Environmental concerns are increasing with the expansion of production and increase in consumption. Thus, the supply chain members arrange to reduce the negative effects resulted from increasing production by using the concept of green supply management which means the purchase, production, distribution, marketing, and management of green products. Madaan and Choudhary (2015) studied product recover system as a strategy to reach sustainability in the supply chain. Zahraee et al. (2018) studied factors which are important in choosing suppliers for green activities in automotive industry in Iran. Kafa et al. (2013) studied the sustainability performance in the management of a supply chain to which the green concept was added for evaluating the sustainability performance. Zhang and Liu (2013) examined the three-stage supply chain that the demand function depends on the product green degree and in this study the green degree is considered as constant. Zhu and He (2017) explored the fact that how green level can be affected by different factors in a supply chain composed of two manufacturers and one retailer. Yang and Xiao (2017) studied a supply chain with the involvement of the government. The supply chain in their study consists of three members. In all three cases, the game theory is used by considering the involvement of government, when the consumer demand and production cost are fuzzy, to study the pricing, level of green, and expected benefit. Madani and Rasti-Barzoki (2017) investigated the government policies for guiding the production toward more green and sustainable production of products. In their study, the benefit of supply chain in case of the financial involvement of the government and product green degree is studied in centralized and decentralized strategies. Mahmoudi and Rasti-Barzoki (2018) considered a green supply chain in which the government looks for optimal decisions to encourage the producers observe the values and control the production of greenhouse gases and global warming. In fact, the financial involvement of government is considered in their supply chain. Basiri and Heydari (2017) studied a supply chain producing a non-green product but attempting to sell a new green product. Jamali et al. (2018) investigated the pricing issue in two competitive supply chains having two sale channels. These two supply chains produce two substitutable products. Jamali and Rasti-Barzoki (2019) studied a sustainable supply chain consists of two manufacturers, produced green and non-green product, and a third-party to examine how the carbon emission and delivery time can be reduced. They found that for gaining a profitable supply chain and acceptable sustainability, it is important to have strategies for competition between members. In the mentioned study, the model is solved in centralized and decentralized games. In this study, the optimal price for both products and optimal green degree for the second product are studied.

As mentioned, the study of supply chain when there is such a risk in the real market became significant due to the presence of demand disruption risk. Todays, green products are highly considered for having the green supply chain. In this study, the supply chain in which the retailer is leader is addressed and the return policy with alliance selection is studied in four strategies. In this chain, it is possible to enter a new potential supplier producing the green product. These factors are studied together to model the real market.

Problem Statement

In this study, the pricing policies, return, and alliance selection are studied in a two-echelon supply chain composed of retailer and supplier. In this supply chain, supplier provides the retailer with his product in wholesale price and the retailer sells it to the customer. Here, there is the possibility of product return both from customer to retailer and from retailer to supplier. In this supply chain, the retailer is the most powerful member having the role of leader and other members should follow him. Since the retailer has the role of leader, he must make decision on whether the new member is allowed to enter or not. For this purpose, first the retailer must consider his profit in the current conditions while the new member enters. If the profit of retailer increases with the arrival of the new member, the retailer will allow the new member enter (No alliance); otherwise, the retailer will not allow (Base mode). Then, if the retailer allows the new member enter the supply chain, the retailer must make decisions on alliance with the new member in four strategies. Retailer (R) has the responsibility of establishing alliance among the members. Supplier (S) and new vendor (V) produce the substitutable products. In addition, here the vendor produces green product while the old supplier produces the non-green product. Another issue that is discussed is to the extent of green degree in case of entering the new vendor. Thus, the following four alliance strategies are studied:

- RV alliance: In this case, the retailer joins the new vendor.
- RS alliance: In this case, the retailer joins the old supplier.
- RSV: In this case, the retailer joins both members.
- No alliance (N): In this case, there is no alliance between the supply chain members.

In addition, the base demand disruption risk is defined in problem definition. Demand cannot be considered constant and definitive in the real world and some factors change the demand. Thus, risk is considered to approach the problem to the real world. The disruption risk can change the demand with a probability. Figures 1 and 2 show the desired supply chain at the base mode and when the new supplier enters.

Model Formulation

The following symbols are used to model the problem. The value of all parameters, decision variables, and functions is between 0 and 1 while the values of profit function are between -1 and $+1$.

Symbols

Parameters:

c_1	cost of the unit of first product by the old supplier
c_2	cost of the unit of second product by the new supplier
ω	The demand replacement rate between the first and second products due to the retail price
θ	The demand replacement rate between the first and second products due to refund amount
γ_1	The demand increase rate due to the refund amount of the first product
γ_2	The demand increase rate due to the refund amount of the second product
F_1	The basic return of the first product which is not dependent on the refund amount
F_2	The basic return of the second product which is not dependent on the refund amount
f_1	return rate of the first product which is dependent on the refund amount
f_2	return rate of the second product which is dependent on the refund amount
λ_1	The sensitivity coefficient of the green degree on the first product demand
λ_2	The sensitivity coefficient of the green degree on the second product demand
μ	The cost-coefficient of the product green degree
k_R^{RV}	The share percent of retailer from the profit obtained from RV alliance
k_V^{RV}	The share percent of new supplier from the profit obtained from RV alliance
k_R^{RS}	The share percent of retailer from the profit obtained from RS alliance
k_V^{RS}	The share percent of new supplier from the profit obtained from RS alliance
k_R^{RSV}	The share percent of retailer from the profit obtained from RSV alliance

Fig. 1 Supply chain before the arrival of the new supplier

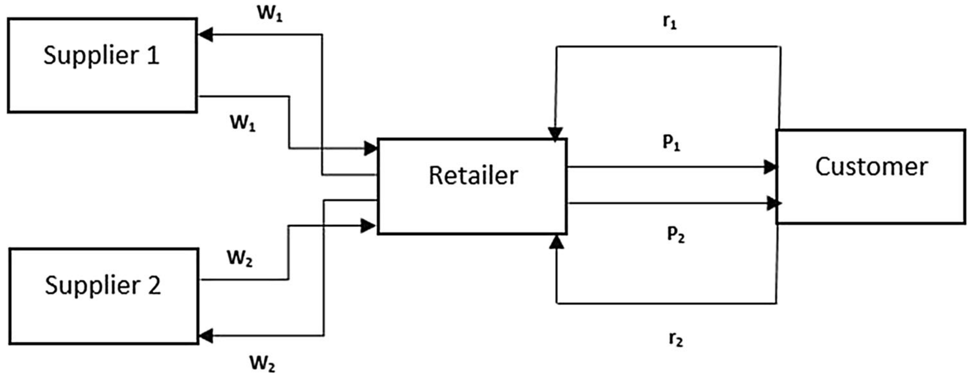

Fig. 2 Supply chain at the presence of the new supplier

k_S^{RSV} The share percent of old supplier from the profit obtained from RSV alliance
k_V^{RSV} The share percent of new supplier from the profit obtained from RSV alliance
Δa_1 The effect of disruption risk on the base demand of the first product
Δa_2 The effect of disruption risk on the base demand of the second product
φ_1 The probability of disruption risk for the first product
φ_2 The probability of disruption risk for the second product

Decision variables:

P_1 The retail price for the first product
P_2 The retail price for the second product
U_1 The favorable margin of revenue obtained from the first product for retailer
U_2 The favorable margin of revenue obtained from the second product for retailer
r_1 The refund amount of the first product
r_2 The refund amount of the second product
W_1 The wholesale price of the first product
W_2 The wholesale price of the second product
θ' The green degree of the second product

Functions:

D_1^B The demand function of the first product at the base mode
R_1^B The return function of the first product at the base mode
D_1 The demand function of the first product without disruption risk
D_2 The demand function of the second product without disruption risk
$\overline{D_1}$ The demand function of the first product with demand disruption risk
$\overline{D_2}$ The demand function of the second product with demand disruption risk
R_1 The return function of the first product
R_2 The return function of the second product
π_R^B The profit function of retailer at the base mode
π_S^B The profit function of old supplier at the base mode
π_R^N The profit function of retailer in no-alliance strategy
π_S^N The profit function of old supplier in no-alliance strategy
π_V^N The profit function of new supplier in no-alliance strategy
π_A^{RS} The profit function of RS alliance
π_V^{RS} The profit function of new supplier in RS alliance
π_A^{RV} The profit function of RV alliance
π_S^{RV} The profit function of old supplier in RV alliance
π_A^{RSV} The profit function of RSV alliance

Base Model

In the base model, no new supplier enters the supply chain. Thus, initially the supply chain is composed of a supplier producing non-green product and a retailer selling his product as wholesale. Demand functions and return amounts for the first product at this strategy are as follows while the demand functions are considered linearly (e.g. Zhou et al. 2015; Taleizadeh et al. 2017). The effect of disruption risk is shown as Δa_1, and the effect of refund amount is considered in demand functions.

$$\overline{D_1^B} = (1 + \Delta a_1) - P_1 + \gamma_1 r_1 \quad (1)$$

$$D_1^{\mathrm{B}} = 1 - P_1 + \gamma_1 r_1 \tag{2}$$

$$R_1^{\mathrm{B}} = F_1 + f_1 r_1 \tag{3}$$

Thus, the profit functions of retailer and supplier at the base mode, where no new supplier has entered the supply chain yet, are as follows while the disruption risk probability is as φ_1:

$$\pi_{\mathrm{R}}^{\mathrm{B}} = (P_1 - W_1)[(1 + \Delta a_1\varphi_1) - P_1 + \gamma_1 r_1] - (r_1 - W_1)R_1^{\mathrm{B}} \tag{4}$$

$$\pi_{\mathrm{S}}^{\mathrm{N}} = (W_1 - c_1)\overline{D_1^{\mathrm{B}}}\varphi_1 + (W_1 - c_1)D_1^{\mathrm{B}}(1 - \varphi_1) - W_1 R_1^{\mathrm{B}} \tag{5}$$

Alliance Strategies

After the arrival of the new supplier to the supply chain, the demand functions for the first non-green product and the second green product in risk and non-risk modes as well as the functions related to return values are as follows:

$$\overline{D_1} = (1 + \Delta a_1) - (1 + \omega)P_1 + \omega P_2 + (\gamma_1 + \theta)r_1 - \theta r_2 - \lambda_1\theta' \tag{6}$$

$$\overline{D_2} = (1 + \Delta a_2) - (1 + \omega)P_2 + \omega P_1 + (\gamma_2 + \theta)r_2 - \theta r_1 + \lambda_2\theta' \tag{7}$$

$$D_1 = 1 - (1 + \omega)P_1 + \omega P_2 + (\gamma_1 + \theta)r_1 - \theta r_2 - \lambda_1\theta' \tag{8}$$

$$D_2 = 1 - (1 + \omega)P_2 + \omega P_1 + (\gamma_2 + \theta)r_2 - \theta r_1 + \lambda_2\theta' \tag{9}$$

$$R_1 = F_1 + f_1 r_1 \tag{10}$$

$$R_2 = F_2 + f_2 r_2 \tag{11}$$

In Eqs. (6)–(9), the effect of green degree for the second product on the demand of the first and second products is shown. In other words, the demand of the first product decreases to $\lambda_1\theta'$ because the green feature of the second product attracts more attention of customer to this product. Thus, the demand of the second product increases to $\lambda_2\theta'$ (Li et al. 2016; Jamali and Rasti-Barzoki 2018). In these equations about demand functions, when there is disruption risk, the base demand value changes as Δa_1 and Δa_2 for the first and second products. It must be noted that the base demand parameter is considered as 1.

No-alliance Strategy

In this strategy, the new supplier enters the supply chain with the approval of the retailer. No alliance is formed between the supply chain members with the decision of the retailer and all members make decisions independently. In this case, the profit functions of each supply chain member are shown as follows:

$$\pi_{\mathrm{S}}^{\mathrm{N}} = (W_1 - c_1)\overline{D_1}\varphi_1 + (W_1 - c_1)D_1(1 - \varphi_1) - W_1 R_1 \tag{12}$$

$$\pi_{\mathrm{V}}^{\mathrm{N}} = (W_2 - c_2)\overline{D_2}\varphi_2 + (W_2 - c_2)D_2(1 - \varphi_2) - W_2 R_2 - \mu\frac{\theta'^2}{2} \tag{13}$$

$$\begin{aligned}\pi_{\mathrm{R}}^{\mathrm{N}} = {} & U_1[(1 + \Delta a_1\varphi_1) - (1 + \omega)P_1 + \omega P_2 + r_1(\gamma_1 + \theta) - \theta r_2 - \lambda_1\theta'] + \\ & U_2[(1 + \Delta a_2\varphi_2) - (1 + \omega)P_2 + \omega P_1 + r_2(\gamma_2 + \theta) - \theta r_1 + \lambda_2\theta'] - (r_1 - W_1)R_1 - (r_2 - W_2)R_2\end{aligned} \tag{14}$$

The production of green product applies extra cost to the new supplier. Thus, another cost as $\mu\frac{\theta'^2}{2}$ is imposed to the new supplier as well as the regular costs for production, as observed in profit function of old supplier in Eq. (12), resulting in reduced profit. Such a reduction is shown in Eq. (13) (Li et al. 2016; Jamali and Rasti-Barzoki 2018; Gao et al. 2016).

RV Alliance

After the arrival of the new member to the supply chain, the retailer may create alliance with the new supplier to obtain the maximum profit. In this case, the profit function of the old supplier and profit function in alliance strategy for the other two members of the chain are as follows:

$$\pi_{\mathrm{S}}^{\mathrm{RV}} = (W_1 - c_1)\overline{D_1}\varphi_1 + (W_1 - c_1)D_1(1 - \varphi_1) - W_1 R_1 \tag{15}$$

$$\begin{aligned}\pi_{\mathrm{A}}^{\mathrm{RV}} = {} & (P_2 - c_2)\overline{D_2}\varphi_2 + (P_2 - c_2)D_2(1 - \varphi_2) \\ & + U_1[(1 + \Delta a_1\varphi_1) - (1 + \omega)P_1 + \omega P_2 + r_1(\gamma_1 + \theta) \\ & - \theta r_2 - \lambda_1\theta'] - r_2 R_2 - (r_1 - W_1)R_1 - \mu\frac{\theta'^2}{2}\end{aligned} \tag{16}$$

RS Alliance

In another strategy, the retailer may establish alliance with the old supplier producing the non-green product to maximize his profit. In this strategy, the profit function for the new supplier and profit function in alliance strategy are shown as follows:

$$\pi_{V}^{RS} = (W_2 - c_2)\overline{D_2}\varphi_2 + (W_2 - c_2)D_2(1 - \varphi_2) - W_2R_2 - \mu\frac{\theta'^2}{2} \tag{17}$$

$$\pi_{A}^{RS} = (P_1 - c_1)\overline{D_1}\varphi_1 + (P_1 - c_1)D_1(1 - \varphi_1) + U_2[(1 + \Delta a_2\varphi_2) - (1 + \omega)P_2 + \omega P_1 + r_2(\gamma_2 + \theta) - \theta r_1 + \lambda_2\theta'] - r_1R_1 - (r_2 - W_2)R_2 \tag{18}$$

RSV Alliance

This alliance strategy is for the time when all three members of the supply chain have an alliance to maximize the profit of retailer. Thus, the profit function in this strategy is calculated as follows:

$$\pi_{A}^{RSV} = (P_1 - c_1)\overline{D_1}\varphi_1 + (P_1 - c_1)D_1(1 - \varphi_1) + (P_2 - c_2)\overline{D_2}\varphi_2 + (P_2 - c_2)D_2(1 - \varphi_2) - r_1R_1 - r_2R_2 - \mu\frac{\theta'^2}{2} \tag{19}$$

Optimal Solutions

After obtaining the profit functions for three members of the supply chain in different strategies of alliance, we solve the proposed model. As mentioned, in this problem the retailer is a leader and other supply chain members follow him while making decisions. When the new supplier enters the supply chain, no alliance is established between the two suppliers, but the formation of such an alliance is performed only by the retailer. In fact, there is a Nash game between the two suppliers.

Solving the Model at the Base Mode

Consider that the retailer in this supply chain is a leader. Due to the Stackelberg game and backward induction method, this model is solved. In this model where no new supplier is added yet to the two old members, first the profit function of the old supplier is optimized to this decision variable by using the backward induction method. Then the values are placed in the profit function of the retailer and the optimal values of his decision variables are calculated. The price of the retailer for the first product is as follows:

$$P_1 = W_1 + U_1 \tag{20}$$

By replacing Eqs. (1), (2), (3), and (20) in Eq. (5) the profit function of the old supplier is as follows:

$$\pi_{S}^{B} = W_1 - c_1 + c_1W_1 - W_1U_1 - W_1^2 - F_1W_1 + \Delta a_1\varphi_1W_1 - c_1\Delta a_1\varphi_1 - f_1W_1r_1 + \gamma_1r_1W_1 - c_1\gamma_1r_1 + c_1U_1 \tag{21}$$

The second derivative of the desired function is used to study the concavity of single variable functions. It can be proved that the profit function in Eq. (21) in W_1 is concave (Proved in "Appendix 1"). Thus, the optimal value of the product wholesale price is calculated as follows:

$$W_1 = 1 - P_1 + r_1\gamma_1 + c_1 - f_1r_1 - F_1 + \Delta a_1\varphi_1 \tag{22}$$

By replacing Eqs. (3) and (22) in Eq. (4), the profit function of retailer in this strategy is as follows:

$$\begin{aligned}\pi_{R}^{B} = {} & 2F_1 - c_1 + 3P_1 + F_1c_1 - 2F_1P_1 - F_1r_1 \\ & - 2\Delta a_1\varphi_1 + c_1P_1 + 2f_1r_1 - 2\gamma_1r_1 - f_1r_1^2 - F_1^2 \\ & - 2P_1^2 - \Delta a_1^2\varphi_1^2 - f_1^2r_1^2 \\ & - \gamma_1^2r_1^2 + 2F_1\Delta a_1\varphi_1 - 2F_1f_1r_1 + 2F_1\gamma_1r_1 \\ & - c_1\Delta a_1\varphi_1 + 3\Delta a_1\varphi_1P_1 + c_1\gamma_1r_1 - c_1\gamma_1r_1 \\ & - 2f_1P_1r_1 + 3\gamma_1P_1r_1 + 2f_1\gamma_1r_1^2 \\ & + 2f_1\Delta a_1\varphi_1r_1 - 2\gamma_1\Delta a_1\varphi_1r_1 - 1\end{aligned} \tag{23}$$

In order to prove the concavity of profit function with two decision variables in Eq. (23), the Hessian matrix is used. The minor of the first order should be negative while the minor of the second order should be positive for concave profit function and such conditions are studied in "Appendix 1". By obtaining the partial derivatives for these two decision variables and solving the obtained equations, P_1 and r_1 are obtained as follows:

$$P_1 = \frac{6f_1 - 2F_1f_1 - 3F_1\gamma_1 + 2c_1f_1 - 2f_1\gamma_1 + 2F_1\gamma_1^2 - c_1\gamma_1^2 + 2f_1^2 - 2F_1f_1\gamma_1 + c_1f_1\gamma_1 + 6f_1\Delta a_1\varphi_1}{4f_1^2 - 4f_1\gamma_1 + 8f_1 - \gamma_1^2} + \frac{2f_1^2\Delta a_1 - 2f_1\gamma_1\Delta a_1\varphi_1}{4f_1^2 - 4f_1\gamma_1 + 8f_1 - \gamma_1^2} \tag{24}$$

$$r_1 = \frac{2f_1 - 4F_1 + \gamma_1 - 4F_1f_1 + 2F_1\gamma_1 + 2c_1f_1 - c_1\gamma_1 + 2f_1\Delta a_1\varphi_1 + \gamma_1\Delta a_1\varphi_1}{4f_1^2 - 4f_1\gamma_1 + 8f_1 - \gamma_1^2} \quad (25)$$

At the base mode, when the new supplier with green product is not entered the supply chain, first the retailer as a leader calculates his decision variables from Eqs. (24) and (25) by Stackelberg game. Then, the old supplier who is the follower of the leader calculates his decision value from Eq. (22) based on the decisions of the retailer.

Solving the No-alliance Strategy

With the arrival of the new member to the supply chain producing the green product, the desired supply chain finds three members including the retailer establishing alliance between the members as a leader, old supplier producing the first and non-green product, and new supplier producing the green product, and this product is a substitute for the first product. In this case, no alliance is established between the supply chain members based on the decision of the retailer. By using the game theory, solving this model is as follows. The price of the second product is considered as follows:

$$P_2 = W_2 + U_2 \quad (26)$$

Based on backward induction method, first two suppliers simultaneously optimize their profit functions and then replace the values in the profit functions of the retailer. Therefore, the decision variables of the retailer are calculated. In order to determine the profit function of the old supplier, Eqs. (6), (8), (10), (20), and (26) are placed in Eq. (12). The following equation is obtained:

$$\begin{aligned}\pi_S^N = {} & W_1 - c_1 - F_1W_1 - bW_1^2 - c_1\Delta a_1\varphi_1 \\ & + bc_1U_1 + bc_1W_1 - c_1\gamma_1r_1 - c_1\omega P_2 + \Delta a_1\varphi_1W_1 \\ & + c_1\lambda_1\theta' - c_1\theta r_1 + c_1\theta r_2 - bU_1W_1 \\ & - f_1r_1W_1 + \gamma_1r_1W_1 + \omega P_2W_1 - \lambda_1\theta'W_1 \\ & - \theta r_1W_1 - \theta r_2W_1\end{aligned} \quad (27)$$

The concavity of Eq. (27) in W_1 is shown in "Appendix 1". Thus, the optimal wholesale price of the product by using the partial derivative is calculated as follows:

$$W_1 = \frac{bc_1 - F_1 + \Delta a_1\varphi_1 - bU_1 - f_1r_1 + \gamma_1r_1 - \lambda_1\theta' + \omega U_2 + \omega W_2 + \theta r_1 - \theta r_2 + 1}{2b} \quad (28)$$

In order to determine the profit function of the new supplier, Eqs. (7), (8), (11), (20), and (26) are replaced in Eq. (13). Then, the desired profit function is obtained as follows:

$$\begin{aligned}\pi_V^N = {} & W_2 - c_2 - F_2W_2 - bW_2^2 - \mu\frac{\theta'^2}{2} - c_2\Delta a_2\varphi_2 \\ & + bc_2U_2 + bc_2W_2 - c_2\gamma_2r_2 \\ & + \Delta a_2\varphi_2W_2 - c_2\lambda_2\theta' - c_2\omega U_1 \\ & - c_2\omega W_1 + c_2\theta r_1 - c_2\theta r_2 - bU_2W_2 - f_2r_2W_2 \\ & + \gamma_2r_2W_2 + \lambda_2\theta'W_2 + \omega U_1W_2 + \omega W_2W_1 \\ & - \theta r_1W_2 + \theta r_2W_2\end{aligned} \quad (29)$$

Based on the conditions considered in "Appendix 1", Eq. (29) in W_2 and Θ' is concave. Thus, the equations for decision variables by using the partial derivatives of functions and solving the equations are as follows:

$$W_2 = \frac{bc_2 - F_2 + \Delta a_2\varphi_2 - bU_2 - f_2r_2 + \gamma_2r_2 + \lambda_2\theta' + \omega U_1 + \omega W_1 + \theta r_2 - \theta r_1 + 1}{2b} \quad (30)$$

$$\theta' = \frac{\lambda_2}{\mu}(W_2 - c_2) \quad (31)$$

By replacing Eqs. (28) and (29) in Eq. (30), the optimal

$$\begin{aligned}W_2 = {} & \frac{2b\mu[1 - F_2 + bc_2 + \omega U_1 - \theta r_1 + \gamma_2r_2 + \theta r_2 - f_2r_2 - bU_2 + \Delta a_2\varphi_2] - 2bc_2\lambda_2^2 + \omega\lambda_1\lambda_2c_2}{\mu(4b^2 - \omega^2) - 2b\lambda_2^2 + \omega\lambda_1\lambda_2} \\ & + \frac{\mu\omega[1 - F_1 + bc_1 + \omega U_2 - \theta r_2 + \gamma_1r_1 + \theta r_1 - f_1r_1 - bU_1 + \Delta a_1\varphi_1]}{\mu(4b^2 - \omega^2) - 2b\lambda_2^2 + \omega\lambda_1\lambda_2}\end{aligned} \quad (32)$$

wholesale price of the second product is calculated as follows:

After simplification, Eq. (38) is obtained based on the following equations:

$$\beta_1 = \frac{2b\mu[1 - F_2 + bc_2 + \Delta a_2\varphi_2] + \mu\omega[1 - F_1 + bc_1 + \Delta a_1\varphi_1] - 2bc_2\lambda_2^2 + \omega\lambda_1\lambda_2 c_2}{\mu(4b^2 - \omega^2) - 2b\lambda_2^2 + \omega\lambda_1\lambda_2} \tag{33}$$

$$\beta_2 = \frac{2b\mu\omega + \mu\omega}{\mu(4b^2 - \omega^2) - 2b\lambda_2^2 + \omega\lambda_1\lambda_2} \tag{34}$$

$$\beta_3 = \frac{\omega + 2b^2\mu}{\mu(4b^2 - \omega^2) - 2b\lambda_2^2 + \omega\lambda_1\lambda_2} \tag{35}$$

$$\beta_4 = \frac{(\gamma_1 + \theta - f_1)\mu\omega - 2b\mu\theta}{\mu(4b^2 - \omega^2) - 2b\lambda_2^2 + \omega\lambda_1\lambda_2} \tag{36}$$

$$\beta_5 = \frac{(\gamma_2 + \theta - f_2)2b\mu - \mu\omega\theta}{\mu(4b^2 - \omega^2) - 2b\lambda_2^2 + \omega\lambda_1\lambda_2} \tag{37}$$

$$W_2 = \beta_1 + \beta_2 U_1 + \beta_3 U_2 + \beta_4 r_1 + \beta_5 r_2 \tag{38}$$

By replacing Eq. (38) in Eq. (31), the optimal value of green degree of the second product is calculated as follows:

$$\theta' = \frac{\lambda_2}{\mu}(\beta_1 + \beta_2 U_1 + \beta_3 U_2 + \beta_4 r_1 + \beta_5 r_2 - c_2) \tag{39}$$

By replacing Eqs. (32) and (33) in Eq. (28), the optimal value of wholesale price for the first product is calculated as follows:

$$\begin{aligned} W_1 = {} & \frac{2b\mu - \lambda_1\lambda_2 + \mu\omega + F_1\lambda_2^2 - \lambda_2^2 + b\lambda_2^2 U_1 + f_1\lambda_2^2 r_2 - 2b^2\mu U_1 - \gamma_1\lambda_2^2 r_1 - \omega\lambda_2^2 U_2 + \omega^2\mu U_1}{\mu(4b^2 - \omega^2) - 2b\lambda_2^2 + \omega\lambda_1\lambda_2} \\ & + \frac{\lambda_2^2\theta r_2 - \lambda_2^2\theta r_1 - 2bF_1\mu + \lambda_1\lambda_2 F_2 - F_2\omega\mu - bc_1\lambda_2^2 + 2b^2 c_1\mu - \Delta a_1\varphi_1\lambda_2^2 - c_2\omega\lambda_2^2 - \Delta a_2\varphi_2\lambda_1\lambda_2}{\mu(4b^2 - \omega^2) - 2b\lambda_2^2 + \omega\lambda_1\lambda_2} \\ & + \frac{\Delta a_2\varphi_2\mu - 2bf_1\mu r_1 + 2b\gamma_1\mu r_1 + b\lambda_1\lambda_2 U_2 + b\omega\mu U_2 + f_2\lambda_1\lambda_2 r_2 - f_2\omega\mu r_2 - \gamma_2\lambda_1\lambda_2 r_2 + \gamma_2\omega\mu r_2}{\mu(4b^2 - \omega^2) - 2b\lambda_2^2 + \omega\lambda_1\lambda_2} \\ & + \frac{2b\mu\theta r_1 - 2b\mu\theta r_2 - \omega\lambda_1\lambda_2 U_1 + \lambda_1\lambda_2\theta r_1 - \lambda_1\lambda_2\theta r_2 - \omega\mu\theta r_1 + \omega\mu\theta r_2 + 2b\Delta a_1\varphi_1\mu + bc_2\lambda_1\lambda_2 + bc_2\omega\mu}{\mu(4b^2 - \omega^2) - 2b\lambda_2^2 + \omega\lambda_1\lambda_2} \end{aligned} \tag{40}$$

Due to the simplification, Eq. (46) is obtained by using the following equations:

$$\alpha_1 = \frac{2b\mu - \lambda_1\lambda_2 + \mu\omega + F_1\lambda_2^2 - \lambda_2^2 - 2bF_1\mu + \lambda_1\lambda_2 F_2 - F_2\omega\mu - bc_1\lambda_2^2 + 2b^2 c_1\mu - \Delta a_1\varphi_1\lambda_2^2 - c_2\omega\lambda_2^2 - \Delta a_2\varphi_2\lambda_1\lambda_2}{\mu(4b^2 - \omega^2) - 2b\lambda_2^2 + \omega\lambda_1\lambda_2} \tag{41}$$

$$\alpha_2 = \frac{b\lambda_2^2 - 2b^2\mu + \omega^2\mu - \omega\lambda_2^2}{\mu(4b^2 - \omega^2) - 2b\lambda_2^2 + \omega\lambda_1\lambda_2} \tag{42}$$

$$\alpha_3 = \frac{b\lambda_1\lambda_2 + b\omega\mu - \omega\lambda_2^2}{\mu(4b^2 - \omega^2) - 2b\lambda_2^2 + \omega\lambda_1\lambda_2} \tag{43}$$

$$\alpha_4 = \frac{f_1\lambda_2^2 - \gamma_1\lambda_2^2 - \lambda_2^2\theta - 2bf_1\mu + 2b\gamma_1\mu + 2b\mu\theta + \lambda_1\lambda_2\theta - \omega\mu\theta}{\mu(4b^2 - \omega^2) - 2b\lambda_2^2 + \omega\lambda_1\lambda_2} \tag{44}$$

$$\alpha_5 = \frac{f_1\lambda_2^2 + \lambda_2^2\theta + f_2\lambda_1\lambda_2 - f_2\omega\mu - \gamma_2\lambda_1\lambda_2 + \gamma_2\omega\mu - 2b\mu\theta - \lambda_1\lambda_2\theta + \omega\mu\theta}{\mu(4b^2 - \omega^2) - 2b\lambda_2^2 + \omega\lambda_1\lambda_2} \tag{45}$$

$$W_1 = \alpha_1 + \alpha_2 U_1 + \alpha_3 U_2 + \alpha_4 r_1 + \alpha_5 r_2 \tag{46}$$

By replacing Eqs. (10), (11), (20), (26), (38), (39), and (46) in Eq. (14), the profit function of the retailer is as follows:

$$U_2 = \det\begin{bmatrix} a_{11} & z_1 & a_{13} & a_{14} \\ a_{21} & z_2 & a_{23} & a_{24} \\ a_{31} & z_3 & a_{33} & a_{34} \\ a_{41} & z_4 & a_{43} & a_{44} \end{bmatrix} \Bigg/ \det\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \tag{50}$$

$$r_1 = \det\begin{bmatrix} a_{11} & a_{12} & z_1 & a_{14} \\ a_{21} & a_{22} & z_2 & a_{24} \\ a_{31} & a_{32} & z_3 & a_{34} \\ a_{41} & a_{42} & z_4 & a_{44} \end{bmatrix} \Bigg/ \det\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \tag{51}$$

$$\begin{aligned}
\pi_R^N = {} & U_1 + U_2 + \alpha_1 F_1 + \beta_1 F_2 - F_2 r_2 - bU_1^2 - bU_2^2 - f_1 r_1^2 - f_2 r_2^2 + \alpha_3\omega U_2^2 + \beta_2\omega U_1^2 + F_1\alpha_4 r_1 + F_1\alpha_5 r_2 + F_2\beta_4 r_1 \\
& + F_2\beta_5 r_2 + F_1\alpha_2 U_1 + F_1\alpha_3 U_2 + F_2\beta_2 U_1 + F_2\beta_3 U_2 - \alpha_1 bU_1 - b\beta_1 U_2 + \alpha_1 f_1 r_1 + \beta_1 f_2 r_2 + \Delta a_1\varphi_1 U_1 + \Delta a_2\varphi_2 U_2 \\
& + \alpha_1\omega U_2 + \beta_1\omega U_1 + \gamma_1 r_1 U_1 + \gamma_2 r_2 U_2 + 2\omega U_1 U_2 + \theta r_1 U_1 - \theta r_1 U_2 - \theta r_2 U_1 + \theta r_2 U_2 - \alpha_2 bU_1^2 - b\beta_3 U_2^2 + \alpha_4 f_1 r_1^2 \\
& + \beta_5 f_2 r_2^2 - \alpha_4 br_1 U_1 - \alpha_5 br_2 U_1 - b\beta_4 r_1 U_2 - b\beta_5 r_2 U_2 + \alpha_5 f_1 r_1 r_2 + \beta_4 f_2 r_1 r_2 - \alpha_3 bU_1 U_2 - \beta_2 bU_1 U_2 + \alpha_2 f_1 r_1 U_1 \\
& + \alpha_3 f_1 r_1 U_2 + \beta_2 f_2 r_2 U_1 + \beta_3 f_2 r_2 U_2 + \alpha_4\omega r_1 U_2 + \alpha_5\omega r_2 U_2 + \beta_4\omega r_1 U_1 + \beta_5\omega r_2 U_1 + \alpha_2\omega U_1 U_2 + \beta_3\omega U_1 U_2 \\
& + \frac{\beta_3\lambda_2^2 U_2^2 + \beta_1\lambda_2^2 U_2 - c_2\lambda_2^2 U_2 - \beta_2\lambda_1\lambda_2 U_1^2 + \beta_4\lambda_2^2 r_1 U_2 + \beta_5\lambda_2^2 r_2 U_2 + \beta_2\lambda_2^2 U_1 U_2 - \beta_1\lambda_1\lambda_2 U_1 + c_2\lambda_1\lambda_2 U_1}{\mu} \\
& - \frac{(\beta_4\lambda_1\lambda_2 r_1 U_1 + \beta_3\lambda_1\lambda_2 U_1 U_2 + \beta_5\lambda_1\lambda_2 r_2 U_1)}{\mu}
\end{aligned} \tag{47}$$

This profit function is concave in U_1, U_2, r_1, and r_2 (Based on the conditions studied in "Appendix 1"). Thus, the obtained equations can be formulated as follows in a matrix by taking partial derivatives from the profit function of the retailer in the related decision variables and placing the obtained equations equal to zero.

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} \tag{48}$$

The optimal values of U_1, U_2, r_1, and r_2 are determined as follows:

$$U_1 = \det\begin{bmatrix} z_1 & a_{12} & a_{13} & a_{14} \\ z_2 & a_{22} & a_{23} & a_{24} \\ z_3 & a_{32} & a_{33} & a_{34} \\ z_4 & a_{42} & a_{43} & a_{44} \end{bmatrix} \Bigg/ \det\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \tag{49}$$

$$r_2 = \det\begin{bmatrix} a_{11} & a_{12} & a_{13} & z_1 \\ a_{21} & a_{22} & a_{23} & z_2 \\ a_{31} & a_{32} & a_{33} & z_3 \\ a_{41} & a_{42} & a_{43} & z_4 \end{bmatrix} \Bigg/ \det\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \tag{52}$$

The retailer obtains U_1, U_2, r_1, and r_2 values from Eqs. (49)–(52). Then, the old and new suppliers calculate the optimal values of W_2, Θ', and W_1 from Eqs. (38), (39), and (46). Finally, the optimal value of the retail price for the first and second products is obtained from Eqs. (20) and (26).

Solving the RV Alliance Strategy

With the arrival of new supplier to the supply chain, the retailer decides to ally with the new supplier. After establishing the alliance between the two above-mentioned members, the old supplier optimizes his profit function independently. Based on Stackelberg game, first the old supplier optimizes his profit function by backward induction method then the profit function resulted from the alliance between the two other members is optimized. For calculating the profit of each member in the alliance, the share percent of retailer and new supplier must be determined. By replacing Eqs. (6), (8), (10), and (20) in Eq. (15), the profit function of the old supplier is as follows:

$$\begin{aligned}\pi_S^{RV} = {} & W_1 - c_1 - F_1 W_1 - bW_1^2 - c_1 \Delta a_1 \varphi_1 + bc_1 U_1 \\ & + bc_1 W_1 - c_1 \gamma_1 r_1 - c_1 \omega P_2 + \Delta a_1 \varphi_1 W_1 \\ & + c_1 \lambda_1 \theta' - c_1 \theta r_1 + c_1 \theta r_2 - bU_1 W_1 \\ & - f_1 r_1 W_1 + \gamma_1 r_1 W_1 + \omega P_2 W_1 - \lambda_1 \theta' W_1 \\ & - \theta r_1 W_1 - \theta r_2 W_1\end{aligned} \tag{53}$$

Equation (53) is concave as shown in "Appendix 1". In order to find the optimal wholesale price of the first product, we take partial derivative of the profit function of the old supplier in Eq. (53). Thus, the optimal wholesale price of the first product is as follows:

$$W_1 = \frac{bc_1 - F_1 + \Delta a_1 \varphi_1 - bU_1 - f_1 r_1 + \gamma_1 r_1 - \lambda_1 \theta' + \omega P_2 + \theta r_1 - \theta r_2 + 1}{2b} \tag{54}$$

By using Eqs. (55)–(58), as shown below, Eq. (54) is simplified and Eq. (59) is obtained:

$$A_1 = \frac{1 + bc_1 - F_1 + \Delta a_1 \varphi_1}{2b} \tag{55}$$

$$A_2 = \frac{\omega}{2b} \tag{56}$$

$$A_3 = \frac{\theta}{2b} \tag{57}$$

$$A_4 = \frac{\gamma_1 + \theta - f_1}{2b} \tag{58}$$

$$W_1 = A_1 + A_2 P_2 - A_3 r_2 + A_4 r_1 - \frac{1}{2} U_1 - \frac{\lambda_1}{2b} \theta' \tag{59}$$

In order to obtain the profit function of RS alliance, Eqs. (7), (9), (10), (11), and (20) are placed in Eq. (16):

$$\begin{aligned}\pi_A^{RV} = {} & P_2 - c_2 + F_1 A_1 - F_1 r_1 - \frac{F_1 U_1}{2} - bP_2^2 - \frac{b}{2} U_2^2 \\ & - f_1 r_1^2 - f_2 r_2^2 - \frac{\mu}{2} \theta'^2 + F_1 A_2 P_2 - F_1 A_3 r_2 \\ & + F_1 A_4 r_1 - c_2 \Delta a_2 \varphi_2 \\ & + bc_2 P_2 - A_1 bU_1 + A_1 f_1 r_1 + \Delta a_2 \varphi_2 P_2 - c_2 \gamma_2 r_2 \\ & + \Delta a_1 \varphi_1 U_1 - c_2 \lambda_2 \theta' - c_2 \omega U_1 - A_1 c_2 \omega + A_1 \omega P_2 \\ & - c_2 \lambda_2 \theta' - \frac{c_2 \omega}{2} U_1 \\ & + \gamma_2 P_2 r_2 + c_2 \theta r_1 - c_2 \theta r_2 - \frac{f_1}{2} r_1 U_1 + \gamma_1 r_1 U_1 \\ & + \lambda_2 P_2 \theta' + \frac{3\omega}{2} P_2 U_1 - \frac{\lambda_1}{2} U_1 \theta' - \theta P_2 r_1 + \theta P_2 r_2 \\ & + \theta U_1 r_1 - \theta U_1 r_2 \\ & + A_4 f_1 r_1^2 + A_4 \omega P_2^2 + A_3 c_2 \omega r_2 + A_4 c_2 \omega r_1 \\ & - A_2 bP_2 U_1 + A_2 f_1 P_2 r_1 + A_3 br_2 U_1 - A_4 br_1 U_1 \\ & - A_3 f_1 r_1 r_2 - A_3 \omega P_2 r_2 \\ & + A_4 \omega P_2 r_1 - \frac{F_1 \lambda_1}{2b} \theta' - A_2 c_2 \omega P_2 + \frac{c_2 \omega \lambda_1}{2b} \theta' \\ & - \frac{f_1 \lambda_1}{2b} r_1 \theta' - \frac{\omega \lambda_1}{2b} P_2 \theta'\end{aligned} \tag{60}$$

The profit function of alliance in Eq. (60) is concave in U_1, P_2, r_1, r_2, and Θ' due to the conditions mentioned in "Appendix 1". The partial derivative is taken from the profit function of Eq. (60) with respect to decision variables. Such derivatives are set equal to zero and then converted as matrix shown below:

$$\begin{bmatrix} a'_{11} & a'_{12} & a'_{13} & a'_{14} & a'_{15} \\ a'_{21} & a'_{22} & a'_{23} & a'_{24} & a'_{25} \\ a'_{31} & a'_{32} & a'_{33} & a'_{34} & a'_{35} \\ a'_{41} & a'_{42} & a'_{43} & a'_{44} & a'_{45} \\ a'_{51} & a'_{52} & a'_{53} & a'_{54} & a'_{55} \end{bmatrix} \begin{bmatrix} U_1 \\ P_2 \\ r_1 \\ r_2 \\ \theta' \end{bmatrix} = \begin{bmatrix} z'_1 \\ z'_2 \\ z'_3 \\ z'_4 \\ z'_5 \end{bmatrix} \tag{61}$$

By solving the above matrix, the optimal values of U_1, P_2, r_1, r_2, and Θ' can be calculated from the following equations:

$$U_1 = \det \begin{bmatrix} z'_1 & a'_{12} & a'_{13} & a'_{14} & a'_{15} \\ z'_2 & a'_{22} & a'_{23} & a'_{24} & a'_{25} \\ z'_3 & a'_{32} & a'_{33} & a'_{34} & a'_{35} \\ z'_4 & a'_{42} & a'_{43} & a'_{44} & a'_{45} \\ z'_5 & a'_{52} & a'_{53} & a'_{54} & a'_{55} \end{bmatrix} \Bigg/ \det \begin{bmatrix} a'_{11} & a'_{12} & a'_{13} & a'_{14} & a'_{15} \\ a'_{21} & a'_{22} & a'_{23} & a'_{24} & a'_{25} \\ a'_{31} & a'_{32} & a'_{33} & a'_{34} & a'_{35} \\ a'_{41} & a'_{42} & a'_{43} & a'_{44} & a'_{45} \\ a'_{51} & a'_{52} & a'_{53} & a'_{54} & a'_{55} \end{bmatrix} \tag{62}$$

$$P_2 = \det\begin{bmatrix} a'_{11} & z'_1 & a'_{13} & a'_{14} & a'_{15} \\ a'_{21} & z'_2 & a'_{23} & a'_{24} & a'_{25} \\ a'_{31} & z'_3 & a'_{33} & a'_{34} & a'_{35} \\ a'_{41} & z'_4 & a'_{43} & a'_{44} & a'_{45} \\ a'_{51} & z'_5 & a'_{53} & a'_{54} & a'_{55} \end{bmatrix} \Bigg/ \det\begin{bmatrix} a'_{11} & a'_{12} & a'_{13} & a'_{14} & a'_{15} \\ a'_{21} & a'_{22} & a'_{23} & a'_{24} & a'_{25} \\ a'_{31} & a'_{32} & a'_{33} & a'_{34} & a'_{35} \\ a'_{41} & a'_{42} & a'_{43} & a'_{44} & a'_{45} \\ a'_{51} & a'_{52} & a'_{53} & a'_{54} & a'_{55} \end{bmatrix} \tag{63}$$

$$r_1 = \det\begin{bmatrix} a'_{11} & a'_{12} & z'_1 & a'_{14} & a'_{15} \\ a'_{21} & a'_{22} & z'_2 & a'_{24} & a'_{25} \\ a'_{31} & a'_{32} & z'_3 & a'_{34} & a'_{35} \\ a'_{41} & a'_{42} & z'_4 & a'_{44} & a'_{45} \\ a'_{51} & a'_{52} & z'_5 & a'_{54} & a'_{55} \end{bmatrix} \Bigg/ \det\begin{bmatrix} a'_{11} & a'_{12} & a'_{13} & a'_{14} & a'_{15} \\ a'_{21} & a'_{22} & a'_{23} & a'_{24} & a'_{25} \\ a'_{31} & a'_{32} & a'_{33} & a'_{34} & a'_{35} \\ a'_{41} & a'_{42} & a'_{43} & a'_{44} & a'_{45} \\ a'_{51} & a'_{52} & a'_{53} & a'_{54} & a'_{55} \end{bmatrix} \tag{64}$$

$$r_2 = \det\begin{bmatrix} a'_{11} & a'_{12} & a'_{13} & z'_1 & a'_{15} \\ a'_{21} & a'_{22} & a'_{23} & z'_2 & a'_{25} \\ a'_{31} & a'_{32} & a'_{33} & z'_3 & a'_{35} \\ a'_{41} & a'_{42} & a'_{43} & z'_4 & a'_{45} \\ a'_{51} & a'_{52} & a'_{53} & z'_5 & a'_{55} \end{bmatrix} \Bigg/ \det\begin{bmatrix} a'_{11} & a'_{12} & a'_{13} & a'_{14} & a'_{15} \\ a'_{21} & a'_{22} & a'_{23} & a'_{24} & a'_{25} \\ a'_{31} & a'_{32} & a'_{33} & a'_{34} & a'_{35} \\ a'_{41} & a'_{42} & a'_{43} & a'_{44} & a'_{45} \\ a'_{51} & a'_{52} & a'_{53} & a'_{54} & a'_{55} \end{bmatrix} \tag{65}$$

$$\theta' = \det\begin{bmatrix} a'_{11} & a'_{12} & a'_{13} & a'_{14} & z'_1 \\ a'_{21} & a'_{22} & a'_{23} & a'_{24} & z'_2 \\ a'_{31} & a'_{32} & a'_{33} & a'_{34} & z'_3 \\ a'_{41} & a'_{42} & a'_{43} & a'_{44} & z'_4 \\ a'_{51} & a'_{52} & a'_{53} & a'_{54} & z'_5 \end{bmatrix} \Bigg/ \det\begin{bmatrix} a'_{11} & a'_{12} & a'_{13} & a'_{14} & a'_{15} \\ a'_{21} & a'_{22} & a'_{23} & a'_{24} & a'_{25} \\ a'_{31} & a'_{32} & a'_{33} & a'_{34} & a'_{35} \\ a'_{41} & a'_{42} & a'_{43} & a'_{44} & a'_{45} \\ a'_{51} & a'_{52} & a'_{53} & a'_{54} & a'_{55} \end{bmatrix} \tag{66}$$

In this case, the retailer and new supplier have alliance with each other and this alliance, as the leader in supply chain, obtains the optimal value of U_1, P_2, r_1, r_2, and Θ' from Eqs. (62)–(66). Then, the old supplier who is the follower calculates the optimal value of his decision from Eq. (59). Then, the old supplier, who is the follower, calculates the optimal value of his decision from Eq. (59). Then, the optimal wholesale price of the first product is obtained from Eq. (20).

Solving the RS Alliance Strategy

RS alliance is another strategy. In order to obtain the profit function of new supplier, Eqs. (7), (9), (11), and (26) are placed in Eq. (17):

$$\begin{aligned}\pi_V^{RS} &= W_2 - c_2 - F_2W_2 - bW_2^2 - \mu\frac{\theta'^2}{2} - c_2\Delta a_2\varphi_2 \\ &+ bc_2U_2 + bc_2W_2 - c_2\gamma_2r_2 - c_2\omega P_1 \\ &+ \Delta a_2\varphi_2W_2 - c_2\lambda_2\theta' \\ &+ c_2\theta r_1 - c_2\theta r_2 - bU_2W_2 - f_2r_2W_2 + \gamma_2r_2W_2 \\ &+ \omega P_1W_2 + \lambda_2\theta'W_2 - \theta r_1W_2 + \theta r_2W_2\end{aligned} \tag{67}$$

Based on conditions studied in "Appendix 1", the profit function is concave in W_2 and Θ'. In order to obtain the optimal value of decision variables, the partial derivatives are set equal to zero. In this case, the following equations are obtained:

$$W_2 = \frac{\mu - c_2\lambda_2^2 - F_2\mu + bc_2\mu + \Delta a_2\varphi_2\mu - b\mu U_2 - f_2\mu r_2 + \gamma_2\mu r_2 + \omega\mu P_1 - \mu\theta r_1 + \mu\theta r_2}{2b\mu - \lambda_2^2} \tag{68}$$

By using Eqs. (69)–(72), the simplified Eq. (73) is obtained:

$$A'_1 = \frac{\mu - c_2\lambda_2^2 - F_2\mu + bc_2\mu + \Delta a_2\varphi_2\mu}{2b\mu - \lambda_2^2} \tag{69}$$

$$A'_2 = \frac{\mu\omega}{2b\mu - \lambda_2^2} \tag{70}$$

$$A'_3 = \frac{\mu\theta}{2b\mu - \lambda_2^2} \tag{71}$$

$$A'_4 = \frac{(\gamma_2 + \theta - f_2)\mu}{2b\mu - \lambda_2^2} \tag{72}$$

$$W_2 = A'_1 + A'_2P_1 - A'_3r_1 + A'_4r_2 - \frac{b\mu}{2b\mu - \lambda_2^2}U_2 \tag{73}$$

$$\theta' = \frac{\lambda_2}{\mu}\left(A'_1 + A'_2P_1 - A'_3r_1 + A'_4r_2 - \frac{b\mu}{2b\mu - \lambda_2^2}U_2 - c_2\right) \tag{74}$$

By replacing Eqs. (6), (8), (10), (11), (26), (73), and (74) in Eq. (18), the profit function of alliance between the old supplier and retailer is obtained:

$$\begin{aligned}\pi_{\mathrm{A}}^{\mathrm{RS}} = {} & P_1 - c_1 + U_2 + A'_1F_2 - F_1r_1 - F_2r_2 - bP_1^2 \\ & - bU_2^2 - f_1r_1^2 - f_2r_2^2 + A'_2F_2P_1 - A'_3F_2r_1 \\ & + A'_4F_2r_2 - c_1\Delta a_1\varphi_1 \\ & + bc_1P_1 + \Delta a_1\varphi_1P_1 - c_1\gamma_1r_1 + \Delta a_2\varphi_2U_2 \\ & - c_1\omega U_2 + \gamma_1P_1r_1 - c_1\theta r_1 + c_1\theta r_2 + \gamma_2r_2U_2 \\ & + 2\omega P_1U_2 + \theta P_1r_1 - \theta P_1r_2 - \theta r_1U_2 \\ & + \theta r_2U_2 + A'_4f_2r_2^2 + A'_2f_2P_1r_2 - A'_3f_2r_1r_2 \\ & - A'_1c_1\omega - A'_1bU_2 + A'_1f_2r_2 + A_1\omega P_1 \\ & + A'_2\omega P_1^2 + A'_3c_1\omega r_1 \\ & - A'_4c_1\omega r_2 - A'_2bP_1U_2 + A'_3br_1U_2 - A'_4br_2U_2 \\ & - A'_3\omega P_1r_1 + A'_4\omega P_1r_2 - A'_2c_1\omega P_1 \\ & - \frac{b\lambda_2^2}{2b\mu - \lambda_2^2}U_2^2 - \frac{A'_2\lambda_1\lambda_2}{\mu}P_1^2 \\ & - \frac{F_2b\mu}{2b\mu - \lambda_2^2}U_2 + \frac{A'_2\lambda_2^2}{\mu}P_1U_2 - \frac{A'_3\lambda_2^2}{\mu}r_1U_2 \\ & + \frac{A'_4\lambda_2^2}{\mu}r_2U_2 - \frac{bf_2\mu r_2U_2}{2b\mu - \lambda_2^2} + \frac{A'_1c_1\lambda_1\lambda_2 - c_2c_1\lambda_1\lambda_2}{\mu} \\ & + \frac{A'_3\lambda_1\lambda_2}{\mu}r_1P_1 \\ & + \frac{(A'_1\lambda_2^2 - c_2\lambda_2^2)}{\mu}U_2 + \frac{\lambda_1\lambda_2(c_2 + A'_2c_1)}{\mu}P_1 - \frac{A'_3c_1\lambda_1\lambda_2r_1}{\mu} \\ & + \frac{A'_4c_1\lambda_1\lambda_2r_2}{\mu} - \frac{A'_4\lambda_1\lambda_2}{\mu}P_1r_2 - \frac{bc_1\lambda_1\lambda_2U_2}{2b\mu - \lambda_2^2} + \frac{bc_1\omega\mu U_2}{2b\mu - \lambda_2^2} \\ & + \frac{(b\lambda_1\lambda_2 - b\omega\mu)}{2b\mu - \lambda_2^2}P_1U_2 \end{aligned} \tag{75}$$

The above-mentioned function is concave in P_1, U_2, r_1, r_2 under the conditions studied in "Appendix 1". First, the partial derivatives of Eq. (75) are taken then the formulations are set equal to zero. Now, we can formulate them as a matrix as follows and obtain the optimal values of decision variables:

$$\begin{bmatrix} a''_{11} & a''_{12} & a''_{13} & a''_{14} \\ a''_{21} & a''_{22} & a''_{23} & a''_{24} \\ a''_{31} & a''_{32} & a''_{33} & a''_{34} \\ a''_{41} & a''_{42} & a''_{43} & a''_{44} \end{bmatrix} \begin{bmatrix} P_1 \\ U_2 \\ r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} z''_1 \\ z''_2 \\ z''_3 \\ z''_4 \end{bmatrix} \tag{76}$$

By solving the matrix in Eq. (76), the optimal values of decision variables are obtained from the following equations:

$$P_1 = \det\begin{bmatrix} z''_1 & a''_{12} & a''_{13} & a''_{14} \\ z''_2 & a''_{22} & a''_{23} & a''_{24} \\ z''_3 & a''_{32} & a''_{33} & a''_{34} \\ z''_4 & a''_{42} & a''_{43} & a''_{44} \end{bmatrix} \Bigg/ \det\begin{bmatrix} a''_{11} & a''_{12} & a''_{13} & a''_{14} \\ a''_{21} & a''_{22} & a''_{23} & a''_{24} \\ a''_{31} & a''_{32} & a''_{33} & a''_{34} \\ a''_{41} & a''_{42} & a''_{43} & a''_{44} \end{bmatrix} \tag{77}$$

$$U_2 = \det\begin{bmatrix} a''_{11} & z''_1 & a''_{13} & a''_{14} \\ a''_{21} & z''_2 & a''_{23} & a''_{24} \\ a''_{31} & z''_3 & a''_{33} & a''_{34} \\ a''_{41} & z''_4 & a''_{43} & a''_{44} \end{bmatrix} \Bigg/ \det\begin{bmatrix} a''_{11} & a''_{12} & a''_{13} & a''_{14} \\ a''_{21} & a''_{22} & a''_{23} & a''_{24} \\ a''_{31} & a''_{32} & a''_{33} & a''_{34} \\ a''_{41} & a''_{42} & a''_{43} & a''_{44} \end{bmatrix} \tag{78}$$

$$r_1 = \det\begin{bmatrix} a''_{11} & a''_{12} & z''_1 & a''_{14} \\ a''_{21} & a''_{22} & z''_2 & a''_{24} \\ a''_{31} & a''_{32} & z''_3 & a''_{34} \\ a''_{41} & a''_{42} & z''_4 & a''_{44} \end{bmatrix} \Bigg/ \det\begin{bmatrix} a''_{11} & a''_{12} & a''_{13} & a''_{14} \\ a''_{21} & a''_{22} & a''_{23} & a''_{24} \\ a''_{31} & a''_{32} & a''_{33} & a''_{34} \\ a''_{41} & a''_{42} & a''_{43} & a''_{44} \end{bmatrix} \tag{79}$$

$$r_2 = \det\begin{bmatrix} a''_{11} & a''_{12} & a''_{13} & z''_1 \\ a''_{21} & a''_{22} & a''_{23} & z''_2 \\ a''_{31} & a''_{32} & a''_{33} & z''_3 \\ a''_{41} & a''_{42} & a''_{43} & z''_4 \end{bmatrix} \Bigg/ \det\begin{bmatrix} a''_{11} & a''_{12} & a''_{13} & a''_{14} \\ a''_{21} & a''_{22} & a''_{23} & a''_{24} \\ a''_{31} & a''_{32} & a''_{33} & a''_{34} \\ a''_{41} & a''_{42} & a''_{43} & a''_{44} \end{bmatrix} \tag{80}$$

In this problem, the retailer is leader in supply chain. Thus, when he establishes an alliance with one of the members, the established alliance is known as the leader of the supply chain. First, from Eqs. (77)–(80) the optimal value of P_1, U_2, r_1, r_2 is calculated. Then, the new supplier who is the follower obtains the optimal values of his decision variables from Eqs. (73) and (74). In addition, the optimal value of P_2 is obtained from Eq. (26).

Solving the RSV Alliance Strategy

In this case, the retailer decides to establish the alliance between three members of supply chain. In addition, all three members of the supply chain must determine their profit share percent including $k_{\mathrm{R}}^{\mathrm{RSV}}$, $k_{\mathrm{S}}^{\mathrm{RSV}}$, and $k_{\mathrm{V}}^{\mathrm{RSV}}$. In order to formulize the profit function in this case, Eqs. (1), (7), (8), (9), (10), and (11) are replaced in Eq. (19). Thus, the profit function in this strategy is as follows:

$$\begin{aligned}\pi_{\mathrm{A}}^{\mathrm{RSV}} &= P_1 - c_1 - c_2 + P_2 - F_1 r_1 - F_2 r_2 - bP_1^2 \\ &\quad - bP_2^2 - f_1 r_1^2 - f_2 r_2^2 - \frac{\mu}{2}\theta'^2 - c_1 \Delta a_1 \varphi_1 - c_2 \Delta a_2 \varphi_2 \\ &\quad + bc_1 P_1 + bc_2 P_2 \\ &\quad + \Delta a_1 \varphi_1 P_1 + \Delta a_2 \varphi_2 P_2 - c_1 \gamma_1 r_1 - c_2 \gamma_2 r_2 \\ &\quad - c_1 \omega P_2 - c_2 \omega P_1 + c_1 \lambda_1 \theta' - c_2 \lambda_2 \theta' + \gamma_1 P_1 r_1 \\ &\quad + \gamma_2 P_2 r_2 - c_1 \theta r_1 + c_1 \theta r_2 + c_2 \theta r_1 \\ &\quad - c_2 \theta r_2 + 2\omega P_1 P_2 - \lambda_1 P_1 \theta' + \lambda_2 P_2 \theta' + \theta P_1 r_1 \\ &\quad - \theta P_2 r_1 + \theta P_2 r_2\end{aligned} \tag{81}$$

The profit function for the alliance in Eq. (81) is concave in P_1, P_2, r_1, r_2, and Θ' (See "Appendix 1"). In order to optimize the profit function in Eq. (81) and obtain the optimal values of decision variables, the partial derivatives of Eq. (81) are taken with respect to P_1, P_2, r_1, r_2, and Θ' and we set the formulations equal to zero, so we can formulate them as a matrix as follows:

$$\begin{bmatrix} a'''_{11} & a'''_{12} & a'''_{13} & a'''_{14} & a'''_{15} \\ a'''_{21} & a'''_{22} & a'''_{23} & a'''_{24} & a'''_{25} \\ a'''_{31} & a'''_{32} & a'''_{33} & a'''_{34} & a'''_{35} \\ a'''_{41} & a'''_{42} & a'''_{43} & a'''_{44} & a'''_{45} \\ a'''_{51} & a'''_{52} & a'''_{53} & a'''_{54} & a'''_{55} \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ r_1 \\ r_2 \\ \theta' \end{bmatrix} = \begin{bmatrix} z'''_1 \\ z'''_2 \\ z'''_3 \\ z'''_4 \\ z'''_5 \end{bmatrix} \tag{82}$$

Thus, the optimal values of decision variables are calculated as follows by solving the above matrix equation:

$$P_1 = \det\begin{bmatrix} z'''_1 & a'''_{12} & a'''_{13} & a'''_{14} & a'''_{15} \\ z'''_2 & a'''_{22} & a'''_{23} & a'''_{24} & a'''_{25} \\ z'''_3 & a'''_{32} & a'''_{33} & a'''_{34} & a'''_{35} \\ z'''_4 & a'''_{42} & a'''_{43} & a'''_{44} & a'''_{45} \\ z'''_5 & a'''_{52} & a'''_{53} & a'''_{54} & a'''_{55} \end{bmatrix} \Bigg/ \det\begin{bmatrix} a'''_{11} & a'''_{12} & a'''_{13} & a'''_{14} & a'''_{15} \\ a'''_{21} & a'''_{22} & a'''_{23} & a'''_{24} & a'''_{25} \\ a'''_{31} & a'''_{32} & a'''_{33} & a'''_{34} & a'''_{35} \\ a'''_{41} & a'''_{42} & a'''_{43} & a'''_{44} & a'''_{45} \\ a'''_{51} & a'''_{52} & a'''_{53} & a'''_{54} & a'''_{55} \end{bmatrix} \tag{83}$$

$$P_2 = \det\begin{bmatrix} a'''_{11} & z'''_1 & a'''_{13} & a'''_{14} & a'''_{15} \\ a'''_{21} & z'''_2 & a'''_{23} & a'''_{24} & a'''_{25} \\ a'''_{31} & z'''_3 & a'''_{33} & a'''_{34} & a'''_{35} \\ a'''_{41} & z'''_4 & a'''_{43} & a'''_{44} & a'''_{45} \\ a'''_{51} & z'''_5 & a'''_{53} & a'''_{54} & a'''_{55} \end{bmatrix} \Bigg/ \det\begin{bmatrix} a'''_{11} & a'''_{12} & a'''_{13} & a'''_{14} & a'''_{15} \\ a'''_{21} & a'''_{22} & a'''_{23} & a'''_{24} & a'''_{25} \\ a'''_{31} & a'''_{32} & a'''_{33} & a'''_{34} & a'''_{35} \\ a'''_{41} & a'''_{42} & a'''_{43} & a'''_{44} & a'''_{45} \\ a'''_{51} & a'''_{52} & a'''_{53} & a'''_{54} & a'''_{55} \end{bmatrix} \tag{84}$$

$$r_1 = \det\begin{bmatrix} a'''_{11} & a'''_{12} & z'''_1 & a'''_{14} & a'''_{15} \\ a'''_{21} & a'''_{22} & z'''_2 & a'''_{24} & a'''_{25} \\ a'''_{31} & a'''_{32} & z'''_3 & a'''_{34} & a'''_{35} \\ a'''_{41} & a'''_{42} & z'''_4 & a'''_{44} & a'''_{45} \\ a'''_{51} & a'''_{52} & z'''_5 & a'''_{54} & a'''_{55} \end{bmatrix} \Bigg/ \det\begin{bmatrix} a'''_{11} & a'''_{12} & a'''_{13} & a'''_{14} & a'''_{15} \\ a'''_{21} & a'''_{22} & a'''_{23} & a'''_{24} & a'''_{25} \\ a'''_{31} & a'''_{32} & a'''_{33} & a'''_{34} & a'''_{35} \\ a'''_{41} & a'''_{42} & a'''_{43} & a'''_{44} & a'''_{45} \\ a'''_{51} & a'''_{52} & a'''_{53} & a'''_{54} & a'''_{55} \end{bmatrix} \tag{85}$$

$$r_2 = \det\begin{bmatrix} a'''_{11} & a'''_{12} & a'''_{13} & z'''_1 & a'''_{15} \\ a'''_{21} & a'''_{22} & a'''_{23} & z'''_2 & a'''_{25} \\ a'''_{31} & a'''_{32} & a'''_{33} & z'''_3 & a'''_{35} \\ a'''_{41} & a'''_{42} & a'''_{43} & z'''_4 & a'''_{45} \\ a'''_{51} & a'''_{52} & a'''_{53} & z'''_5 & a'''_{55} \end{bmatrix} \Bigg/ \det\begin{bmatrix} a'''_{11} & a'''_{12} & a'''_{13} & a'''_{14} & a'''_{15} \\ a'''_{21} & a'''_{22} & a'''_{23} & a'''_{24} & a'''_{25} \\ a'''_{31} & a'''_{32} & a'''_{33} & a'''_{34} & a'''_{35} \\ a'''_{41} & a'''_{42} & a'''_{43} & a'''_{44} & a'''_{45} \\ a'''_{51} & a'''_{52} & a'''_{53} & a'''_{54} & a'''_{55} \end{bmatrix} \tag{86}$$

$$\theta' = \det\begin{bmatrix} a'''_{11} & a'''_{12} & a'''_{13} & a'''_{14} & z'''_1 \\ a'''_{21} & a'''_{22} & a'''_{23} & a'''_{24} & z'''_2 \\ a'''_{31} & a'''_{32} & a'''_{33} & a'''_{34} & z'''_3 \\ a'''_{41} & a'''_{42} & a'''_{43} & a'''_{44} & z'''_4 \\ a'''_{51} & a'''_{52} & a'''_{53} & a'''_{54} & z'''_5 \end{bmatrix} \Bigg/ \det\begin{bmatrix} a'''_{11} & a'''_{12} & a'''_{13} & a'''_{14} & a'''_{15} \\ a'''_{21} & a'''_{22} & a'''_{23} & a'''_{24} & a'''_{25} \\ a'''_{31} & a'''_{32} & a'''_{33} & a'''_{34} & a'''_{35} \\ a'''_{41} & a'''_{42} & a'''_{43} & a'''_{44} & a'''_{45} \\ a'''_{51} & a'''_{52} & a'''_{53} & a'''_{54} & a'''_{55} \end{bmatrix} \tag{87}$$

The optimal values of decision variables are calculated from Eqs. (83)–(87).

Decision-Making Stages of Optimal Alliance Strategy Selection

At sections (4) and (5), some models are presented for all strategies of alliance strategy and the optimal solutions for solving these models are studied. In the desired supply chain, a new supplier with a green product intends to enter the two-echelon supply chain composed of a supplier and a dominant retailer. In order to make decision in this strategy, the following procedure is used:

First, the profit function of retailer is calculated when no new supplier is not entered yet to the supply chain. Then, the profit of retailer is calculated by using the above-mentioned equations when the new member is added to the supply chain, but no alliance is established between the members. If the profit of retailer in both strategies is positive and the profit of retailer is more, when no new member is added to the supply chain, the new member is not allowed to enter. Otherwise, the total profit obtained from the alliance is calculated from Eqs. (16), (18), and (19) for three strategies of alliance (Π_{A}^{RV}, Π_{A}^{RS}, and Π_{A}^{RSV}). Then, the profit of each alliance member is obtained for each supply chain member. Alliance strategies are sorted in a descending order based on the profit of retailer. If the profit of retailer is the same in two or several strategies, these strategies will be sorted in a descending order based on the total profit of the supply chain. Accordingly, if the profit of retailer and at least one of the other supply chain members is greater than zero at the first level of ranking, this level of strategy will be selected. Otherwise, the next level will be studied. If none of the ranked levels is consistent with the above-mentioned conditions, no alliance strategy will be selected as the optimal alliance strategy.

Numerical Example

In this study, the parameters used in the model are shown in Table 1 for the numerical analysis of the problem. First, all model parameters must have a value between zero and one. In addition, the return value for both products must be less than the demand value in risky and non-risky modes ($R_1 \leq \text{Min}\{D_1, \overline{D_1}\}, R_2 \leq \text{Min}\{D_2, \overline{D_2}\}$).

Regarding the parameters value in Table 1 and the obtained equations at section (4), the optimal values of decision variables and profit function values at the base mode are calculated. The results are shown in Table 2. Then, the decision variables and profit values for each member of the supply chain are calculated by using the obtained equations for decision strategy. Now, the optimal alliance strategy is discussed due to the obtained results.

Comparing the profit functions of retailer at the base mode and when the new supplier has entered the supply chain without, no alliance shows that the arrival of a new member increases the profit of retailer. Thus, the retailer allows the new member enters the supply chain. Based on the profit values calculated in Table 3 and percent of profit sharing in Table 1, ranking the alliance strategies is calculated in Table 4. Accordingly, RV alliance strategy allocating the first rank in Table 4 is selected as the optimal strategy.

The obtained results indicate that any alliance between the supply chain members increases the total profit of the supply chain. However, in the studied model, the main criterion for selecting any optimal alliance is the profit of retailer. Table 4 shows the optimal profit of all three members in RS and RSV strategies while it is not selected as optimal strategy because the profit of retailer in RV strategy is more than these two strategies. Thus, this strategy is selected as the optimal strategy

Sensitivity Analysis

In order to study the effect of changing some parameters on the type of selected optimal strategy, the green degree and effect of disruption risk, first some rules are considered. These are as follows:

1. If the value of the obtained decision variables is more than 1, these variables will be considered as 1.
2. If the optimal value of return is negative, this value will be considered as zero.
3. If the optimal value of refund amount is more than the retail optimal value, the optimal value of refund amount will be equal to the optimal value of the retail price.

In the obtained tables, the bold parts indicate the use of these rules.

The details of the conducted analysis are shown in some Tables in "Appendix 2". Based on Fig. 3, the effect of changing the parameter λ_2 on green degree can be observed. This figure shows that the green degree for all mentioned strategies increases with the increase in this parameter. As a result, the effect of this increase in RV strategy for green degree is more. This conclusion is observed in the study of Yang and Xiao (2017). In other words, the effect of green degree in demand increases. Thus, the green degree increases to have more demand. Furthermore, by observing Fig. 3, it can be understood that the green degree has its highest value in no alliance strategy when λ_2 is equal to 0.01, while in other alliance strategies the highest value of the green degree is in RV strategy. As shown in Fig. 4, the green degree reduces in all alliance strategies with the increase in this parameter. In this case, the decrease in the green degree of the second product in RV and RSV strategies are greater than other strategies. However, according to Tables 5 and 6, the changes of these two parameters cause no change in the selected optimal strategy.

Figure 5 shows the effect of changing the parameter μ on green degree. Thus, the green degree decreases with the increase in this cost-coefficient. In other words, the costs for producing the green product increase with the increase in this parameter. Thus, the new supplier has to select less green degree for his product to reduce the costs and compete with the first product. In addition, as shown in Table 7, the profit of new supplier producing the green product reduces with the increase in this parameter while the profit of the other two members in all alliance strategies increases. However, no change is created in the type of selected optimal strategy.

Based on Fig. 6, showing the effect of changing the probability of disruption risk occurrence, it can be observed

Table 1 The parameter values of the model

Parameter	Value	Parameter	Value
ω	0.7	λ_2	0.02
Θ	0.05	μ	0.1
c_1	0.4	$\Delta a_1 = \Delta a_2$	0.2
c_2	0.15	k_R^{RV}	0.7
$\gamma_1 = \gamma_2$	0.4	k_R^{RV}	0.3
$F_1 = F_2$	0.1	k_R^{RS}	0.7
$f_1 = f_2$	0.11	k_S^{RS}	0.3
$\varphi_1 = \varphi_2$	0.1	k_R^{RSV}	0.7
λ_1	0.01	$k_S^{RSV} = k_V^{RSV}$	0.15

Table 2 The results of the base mode (B)

Demand		Return	Decision variables			Profit		Total profit of the chain
$\bar{D}_1$	D_1	R_1	W_1	P_1	r_1	R	S	
0.4364	0.2364	0.1365	0.5199	0.8962	0.3315	0.1222	− 0.04021	0.152

Table 3 The results at any alliance strategy

	Alliance strategy	N	RV	RS	RSV
Demand	D_1	0.1757	0.23145	0.3327	0.22557
	$\bar{D}_1$	0.3757	0.43145	0.5327	0.32557
	D_2	0.4275	0.6463	0.363	0.61997
	$\bar{D}_2$	0.6275	0.8463	0.563	0.71997
Return	R_1	0.1475	0.15112	0.122	0.11801
	R_2	0.147	0.16338	0.1486	0.1591
Decision variables	W_1	0.6057	0.41258	–	–
	W_2	0.4492	–	0.2879	–
	U_1	0.3174	0.43146	–	–
	U_2	0.3689	–	0.5117	–
	P_1	0.9231	0.84404	0.7615	0.7606
	P_2	0.8181	0.6953	0.7995	0.6749
	r_1	0.4316	0.46474	0.1999	0.53724
	r_2	0.4276	0.57621	0.4414	0.53724
	Θ’	0.05984	0.07181	0.02757	0.068927
Profit	A	–	0.36955	0.2763	0.31061
	R	0.256	–	–	–
	S	− 0.04907	-0.059186	–	–
	V	0.06765	–	0.0099	–
	Chain profit	0.27458	0.310354	0.2862	0.31061

that the total profit of the supply chain increases with the increase in risk occurrence while RV strategy is still used as the optimal strategy. Figure 7 shows the effect of changing the cost of the first product. The total profit of the supply chain in all strategies reduces with the increase in this cost while this parameter causes a change in the

Table 4 Ranking based on the net profit of each member in every selection strategy

Alliance strategy	Net profit of each member of the supply chain			Rank
	R	S	V	
RV	0.258678	− 0.059186	0.110862	1
N	0.256	− 0.04907	0.06765	2
RS	0.19341	0.08289	0.0099	3
RSV	0.217427	0.0465915	0.0465915	4

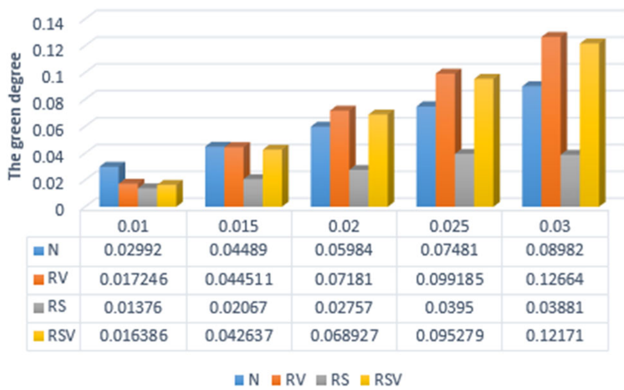

Fig. 3 The effect of changing parameter λ_2 on green degree

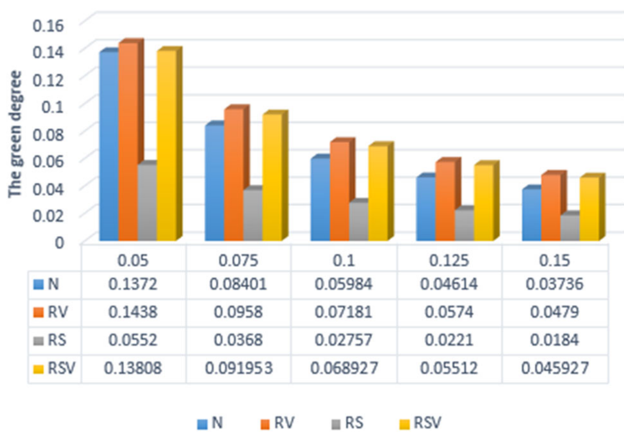

Fig. 4 The effect of changing parameter λ_1 on green degree

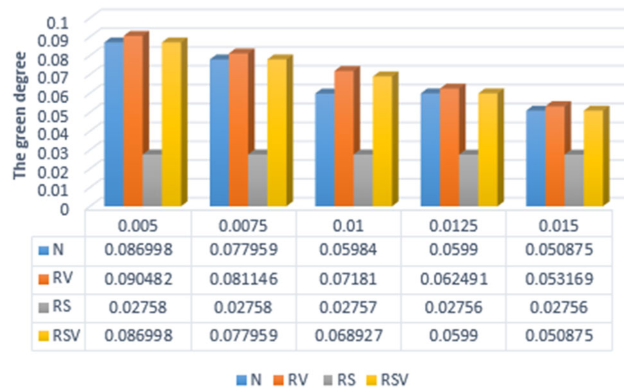

Fig. 5 The effect of changing the parameter μ on green degree

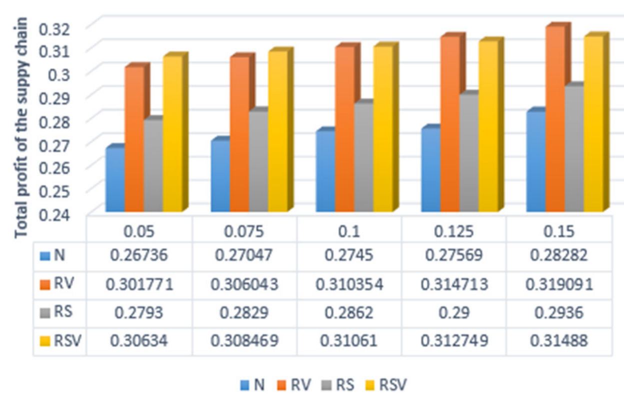

Fig. 6 The effect of changing the probability of disruption risk occurrence on the total profit of the supply chain

optimal strategy. This result is observed in the study of Taleizadeh et al. (2017). Based on Table 9, the no-alliance strategy will be selected as the optimal strategy if the value of this parameter is 0.2 and 0.3.

Figure 8 shows that with the increase in parameter c_1 the green degree increases. In other words, the new supplier increases the green degree for his product in order to remain in the market, compare to it, and attract the customer because price and cost do not only matter to the customers but other factors such as green degree are of great significance.

The effect of changing the parameter f_1 in all strategies can be observed in Table 10. Based on the obtained results, this parameter causes some changes in the optimal strategy. Based on the mentioned table, when f_1 is equal to 0.165, the no-alliance strategy is selected as the optimal one. However, changing the parameter f_1 can have effect on the selected optimal alliance strategy, in Fig. 9, it can be observed that this parameter has a little effect on the green degree and its impact on the mentioned decision variable is greater in no alliance strategy than others.

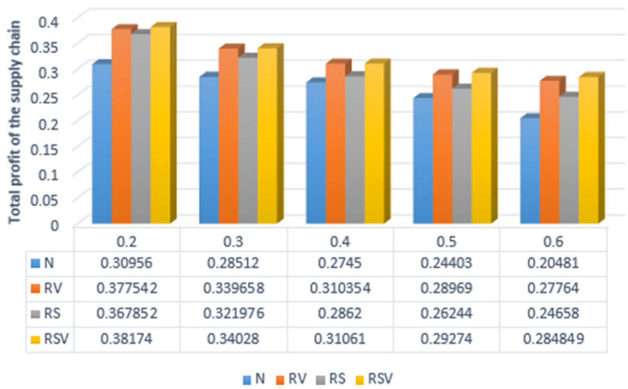

Fig. 7 The effect of changing the parameter c_1 on the total profit of the supply chain

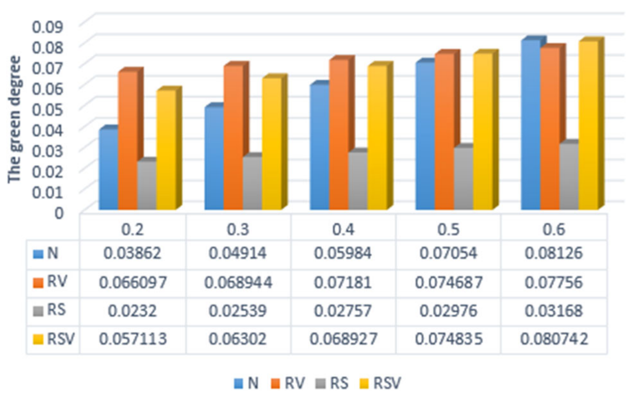

Fig. 8 The effect of changing the parameter c_1 on green degree of products

In Fig. 10, axis X represents parameter λ_2, axis Y represents parameter μ, and axis Z shows the green degree of the second product. The green degree in all strategies has the maximum amount when parameter λ_2 is maximum and parameter μ is minimum. In addition, the green degree in RV alliance strategy is maximum than the other strategies. In RV, RS, and RSV alliances, the green degree is more when the two parameters are maximum than the case the two parameters are minimum. The opposite is true in no-alliance strategy. The details of calculations can be observed in Table 11.

Conclusion

With the increasing significance of effective supply chain, companies seek to select different approaches for achieving this goal. In addition, the production of green products is another factor strongly discussed in production due to the increasing concerns in relation to the environment. Demand is always fluctuating in the real world. Thus, this demand fluctuation is simultaneously discussed in the studies along

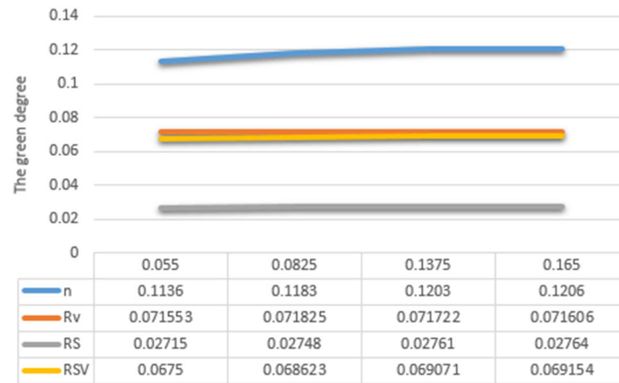

Fig. 9 The effect of changing the parameter f_1 on green degree of products

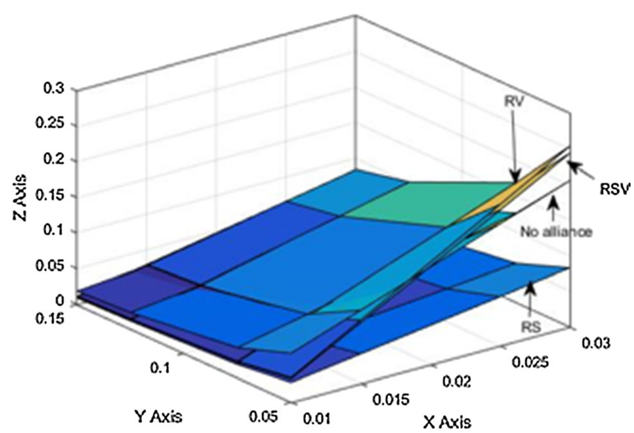

Fig. 10 The effect of simultaneous change of λ_2 and μ on green degree

with the other factors to approach the presented models to the reality. In today's market, new members may enter the supply chain producing the substitutable products of the existing products. Thus, the arrival of upstream members is an important factor that should be considered in the supply chain. In this regard, the supply chain is discussed when the retailer has the role of leader and faces the challenge that whether he should allow the arrival of the substitutable green product to the first product or not. As observed, the arrival of the new supplier to the supply chain is studied in all alliance strategies in favor of all supply chain members. In addition, the optimal alliance strategies among all supply chain members are studied by solving the presented models

Based on the obtained results, creating a centralized supply chain is not always beneficial for all supply chain members. Hence, different alliance strategies must be studied for obtaining an effective and profitable supply chain. Based on the conducted study in this desired supply chain, the cost production of the first product and parameter f_1 cause a change in the optimal alliance strategy.

However, it does not mean that other parameters are ineffective in the model, but the effect of changing the parameters on the profit of the supply chain members can be observed due to the results shown in the tables. On the other hand, we displayed that the changing of parameter f_1 does not have a significant effect on the value of the green degree. Moreover, the increase in the cost of production of the first product increases the green degree. The production of green products applies different costs on the manufacturer. Therefore, the optimal value of the green degree is analyzed.

As shown, the increasing sensitivity coefficient of the green degree increased the green degree of the second product that is more in RV alliance, so that when this sensitivity coefficient equals to 0.03, the green degree of the second product increases to 0.12664. The increase in sensitivity coefficient of green degree in RS is very low. In addition, when λ_2 is equal to 0.01, the green degree has its highest value in no alliance strategy, but in other alliance strategies the highest value of the green degree is in RV strategy. Furthermore, the increased cost-coefficient of green degree decreases the green degree. This sensitivity to the increasing cost-coefficient of the green degree in RS alliance is lower compared to other alliance strategies. Lastly, we showed the effect of simultaneous change of λ_2 and μ on the green degree. We observed the lowest and highest value of the green degree in all kinds of alliance strategies and noticed their differences among strategies. Consequently, the proposed model comprised of different factors investigates the real situations in the market and provides numerical example and sensitivity analysis to give understandable insights.

For having an effective supply chain, managers must formulate their model. By using the proposed model in this study, it can be seen that, forming an alliance is not beneficial all the time for members. It means that managers must make decision based on the profit of the members. When alliance is formed, it is shown that changing the cost of the production and parameter f_1 can change the optimal alliance strategy. Thus, return rate of the non-green product changes the optimal alliance strategy. Additionally, it can be seen that the green degree is various in 4 types of alliance strategies. On the other hand, In RV, RS, and RSV alliances, the green degree is more when λ_2 and μ are maximum than the case the two parameters are minimum. The opposite is true in no-alliance strategy. In addition, arrival of the new green producer in not beneficial for the retailer all the time; therefore, by formulating the real situation the optimal decision can be made.

Our study can be extended by considering deadline for return products. This model is studied in single period; thus it can be studied in multi-period phase. In this study online channel is ignored. As a result, studying the supply chain when there is a chance for both online and traditional channel would be interesting to investigate. Adding cost disruption risk to the model can make our model more realistic. In general, the problem statement can extend to other industries like medical supply chain.

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Compliance with Ethical Standards

Conflict of interest The authors hereby declare that there are no potential conflicts of interest in terms of authorship, research and/or publication of this article.

Appendix 1

Proving the concavity of Eq. (21):

$$\frac{\partial^2 \pi_S^B}{\partial W_1^2} = -2 \leq 0 \tag{88}$$

Thus, the desired equation is concave in the decision variable.

Proving the concavity of Eq. (23):

$$\frac{\partial^2 \pi_R^B}{\partial P_1^2} = -4 \leq 0 \tag{89}$$

$$\det \begin{bmatrix} \frac{\partial^2 \pi_R^B}{\partial P_1^2} & \frac{\partial^2 \pi_R^B}{\partial P_1 \partial r_1} \\ \frac{\partial^2 \pi_R^B}{\partial r_1 \partial P_1} & \frac{\partial^2 \pi_R^B}{\partial r_1^2} \end{bmatrix} \geq 0 \tag{90}$$

Proving the concavity of Eq. (27):

$$\frac{\partial^2 \pi_S^N}{\partial W_1^2} = -2b \leq 0 \tag{91}$$

Thus, Eq. (91) is concave in the decision variable.

Proving the concavity of Eq. (29):

The following conditions must be established for the concavity of Eq. (29):

$$\frac{\partial^2 \pi_V^N}{\partial W_2^2} = -2b \leq 0 \tag{92}$$

$$\det \begin{pmatrix} -2b & \lambda_2 \\ \lambda_2 & -\mu \end{pmatrix} = 2b\mu - \lambda_2^2 \geq 0 \tag{93}$$

Equation (92) is always established and the established Eq. (93) must be studied.

Proving the concavity of Eq. (47):

$$a_{11} \leq 0 \tag{94}$$

$$\det\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \geq 0 \tag{95}$$

$$\det\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \leq 0 \tag{96}$$

$$\det\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \geq 0 \tag{97}$$

The four above-mentioned conditions must be established for the concavity of Eq. (47). Obviously, the elements of the above-mentioned matrices are as follows:

$$a_{11} = \frac{\partial^2 \pi_R^N}{\partial U_1^2},\ a_{12} = \frac{\partial^2 \pi_R^N}{\partial U_1 \partial U_2},\ a_{13} = \frac{\partial^2 \pi_R^N}{\partial U_1 \partial r_1},\ a_{14} = \frac{\partial^2 \pi_R^N}{\partial U_1 \partial r_2}, \ldots, a_{44} = \frac{\partial^2 \pi_R^N}{\partial r_2^2} \tag{98}$$

Proving the concavity of Eq. (53):

$$\frac{\partial^2 \pi_S^{RV}}{\partial W_1^2} = -2b < 0 \tag{99}$$

Thus, Eq. (99) is concave in the decision variable.

Proving the concavity of Eq. (60):

Equation (60) is concave in U_1, P_2, r_1, r_2, and Θ' when the following conditions are established:

$$a'_{11} \tag{100}$$

$$\det\begin{bmatrix} a'_{11} & a'_{12} \\ a'_{21} & a'_{22} \end{bmatrix} \geq 0 \tag{101}$$

$$\det\begin{bmatrix} a'_{11} & a'_{12} & a'_{13} \\ a'_{21} & a'_{22} & a'_{23} \\ a'_{31} & a'_{32} & a'_{33} \end{bmatrix} \leq 0 \tag{102}$$

$$\det\begin{bmatrix} a'_{11} & a'_{12} & a'_{13} & a'_{14} \\ a'_{21} & a'_{22} & a'_{23} & a'_{24} \\ a'_{31} & a'_{32} & a'_{33} & a'_{34} \\ a'_{41} & a'_{42} & a'_{43} & a'_{44} \end{bmatrix} \geq 0 \tag{103}$$

$$\det\begin{bmatrix} a'_{11} & a'_{12} & a'_{13} & a'_{14} & a'_{15} \\ a'_{21} & a'_{22} & a'_{23} & a'_{24} & a'_{25} \\ a'_{31} & a'_{32} & a'_{33} & a'_{34} & a'_{35} \\ a'_{41} & a'_{42} & a'_{43} & a'_{44} & a'_{45} \\ a'_{51} & a'_{52} & a'_{53} & a'_{54} & a'_{55} \end{bmatrix} \leq 0 \tag{104}$$

The elements of the matrices are as follows:

$$a'_{11} = \frac{\partial^2 \pi_S^{RV}}{\partial U_1^2},\ a'_{12} = \frac{\partial^2 \pi_S^{RV}}{\partial U_1 \partial P_2},\ a'_{13} = \frac{\partial^2 \pi_S^{RV}}{\partial U_1 \partial r_1},\ a'_{14} = \frac{\partial^2 \pi_S^{RV}}{\partial U_1 \partial r_2},\ a'_{15} = \frac{\partial^2 \pi_S^{RV}}{\partial U_1 \partial \theta'}, \ldots, a'_{55} = \frac{\partial^2 \pi_S^{RV}}{\partial \theta'^2} \tag{105}$$

Proving the concavity of Eq. (67):

$$\frac{\partial^2 \pi_V^{RS}}{\partial W_2^2} = -2b < 0 \tag{106}$$

$$\det\begin{pmatrix} -2b & \lambda_2 \\ \lambda_2 & -\mu \end{pmatrix} = 2b\mu - \lambda_2^2 \geq 0 \tag{107}$$

Equation (106) is always established while the establishment of Eq. (107) must be studied. In case of correct establishment, it can be said that Eq. (67) is concave in W_2 and Θ'.

Proving the concavity of Eq. (75):

$$a''_{11} \leq 0 \tag{108}$$

$$\det\begin{bmatrix} a''_{11} & a''_{12} \\ a''_{21} & a''_{22} \end{bmatrix} \geq 0 \tag{109}$$

$$\det\begin{bmatrix} a''_{11} & a''_{12} & a''_{13} \\ a''_{21} & a''_{22} & a''_{23} \\ a''_{31} & a''_{32} & a''_{33} \end{bmatrix} \leq 0 \tag{110}$$

$$\det\begin{bmatrix} a''_{11} & a''_{12} & a''_{13} & a''_{14} \\ a''_{21} & a''_{22} & a''_{23} & a''_{24} \\ a''_{31} & a''_{32} & a''_{33} & a''_{34} \\ a''_{41} & a''_{42} & a''_{43} & a''_{44} \end{bmatrix} \geq 0 \tag{111}$$

The above conditions must be established for concavity of Eq. (75).

Proving the concavity of Eq. (81):

For proving the concavity of Eq. (81) in P_1, P_2, r_1, r_2, and Θ' the following conditions must be studied and the function in P_1, P_2, r_1, r_2, and Θ' is concave when the following conditions are established.

$$a'''_{11} \leq 0 \tag{112}$$

$$\det\begin{bmatrix} a'''_{11} & a'''_{12} \\ a'''_{21} & a'''_{22} \end{bmatrix} \geq 0 \tag{113}$$

$$\det\begin{bmatrix} a'''_{11} & a'''_{12} & a'''_{13} \\ a'''_{21} & a'''_{22} & a'''_{23} \\ a'''_{31} & a'''_{32} & a'''_{33} \end{bmatrix} \leq 0 \tag{114}$$

$$\det\begin{bmatrix} a'''_{11} & a'''_{12} & a'''_{13} & a'''_{14} \\ a'''_{21} & a'''_{22} & a'''_{23} & a'''_{24} \\ a'''_{31} & a'''_{32} & a'''_{33} & a'''_{34} \\ a'''_{41} & a'''_{42} & a'''_{43} & a'''_{44} \end{bmatrix} \geq 0 \tag{115}$$

$$\det\begin{bmatrix} a'''_{11} & a'''_{12} & a'''_{13} & a'''_{14} & a'''_{15} \\ a'''_{21} & a'''_{22} & a'''_{23} & a'''_{24} & a'''_{25} \\ a'''_{31} & a'''_{32} & a'''_{33} & a'''_{34} & a'''_{35} \\ a'''_{41} & a'''_{42} & a'''_{43} & a'''_{44} & a'''_{45} \\ a'''_{51} & a'''_{52} & a'''_{53} & a'''_{54} & a'''_{55} \end{bmatrix} \leq 0 \tag{116}$$

Appendix 2

See Tables 5, 6, 7, 8, 9, 10 and 11.

Table 5 The effect of changing the parameter λ_1 on green degree

λ_1	No alliance				RV alliance				RS alliance				RSV alliance			
	π_R	π_S	π_V	θ'	π_R	π_S	π_V	θ'	π_R	π_S	π_V	θ'	π_R	π_S	π_V	θ'
0.005	0.256	− 0.0491	0.06768	0.05984	0.25879	− 0.0592	0.11091	0.090482	0.19348	0.08292	0.01	0.02758	0.217225	0.0466125	0.0466125	0.086998
0.0075	0.256	− 0.0491	0.06768	0.05985	0.25873	− 0.0591	0.11089	0.081146	0.19341	0.08289	0.01	0.02758	0.217469	0.0466005	0.0466005	0.077959
0.0125	0.256	− 0.0491	0.06764	0.05985	0.258636	− 0.0592	0.11084	0.062491	0.19341	0.08289	0.01	0.02756	0.217385	0.0465825	0.0465825	0.0599
0.015	0.256	− 0.0491	0.06764	0.05986	0.258601	− 0.0592	0.11083	0.053169	0.19341	0.08289	0.01	0.02756	0.217357	0.0465765	0.0465765	0.050875

Table 6 The effect of changing the parameter λ_2 on green degree

λ_2	RSV alliance				RV alliance				RS alliance				RSV alliance			
	π_R	π_S	π_V	θ'	π_R	π_S	π_V	θ'	π_R	π_S	π_V	θ'	π_R	π_S	π_V	θ'
0.01	0.256	− 0.0491	0.06757	0.02992	0.25851	− 0.0592	0.11079	0.017246	0.19334	0.08286	0.01	0.01376	0.21721	0.04659	0.04659	0.016386
0.015	0.256	− 0.0491	0.0676	0.04489	0.258573	− 0.0592	0.110817	0.044511	0.19334	0.08286	0.01	0.02067	0.217329	0.0465705	0.0465705	0.042637
0.025	0.256	− 0.0491	0.06769	0.07481	0.258846	− 0.0592	0.110934	0.099185	0.19345	0.08292	0.01	0.0395	0.217574	0.046623	0.046623	0.095279
0.03	0.256	− 0.0492	0.06774	0.08982	0.259063	− 0.0592	0.111027	0.12664	0.19355	0.08295	0.01	0.03881	0.21777	0.046665	0.046665	0.12171

Table 7 The effect of changing the parameter μ on green degree

μ	No alliance				RV alliance				RS alliance				RSV alliance			
	π_R	π_S	π_V	θ'	π_R	π_S	π_V	θ'	π_R	π_S	π_V	θ'	π_R	π_S	π_V	θ'
0.05	0.2483	− 0.0905	0.08165	0.1372	0.25886	− 0.059195	0.11094	0.1438	0.19348	0.08292	0.01	0.0552	0.217588	0.046626	0.046626	0.13808
0.075	0.2477	− 0.0609	0.07781	0.08401	0.258741	− 0.059189	0.110889	0.0958	0.19341	0.08289	0.01	0.0368	0.217483	0.0466035	0.0466035	0.091953
0.125	0.263	− 0.0463	0.06049	0.04614	0.258643	− 0.059185	0.110847	0.0574	0.19334	0.08286	0.01	0.0221	0.217392	0.046584	0.046584	0.05512
0.15	0.269	− 0.0463	0.05519	0.03736	0.258662	− 0.059184	0.110838	0.0479	0.19334	0.08286	0.01	0.0184	0.217371	0.0465795	0.0465795	0.045927

Table 8 The effect of changing the probability of disruption risk on the total profit of the supply chain

$\varphi_1 = \varphi_2$	No alliance				RV alliance				RS alliance				RSV alliance			
	π_R	π_S	π_V	θ'	π_R	π_S	π_V	θ'	π_R	π_S	π_V	θ'	π_R	π_S	π_V	θ'
0.05	0.251	− 0.04879	0.06515	0.0596	0.252315	− 0.058679	0.108135	0.070865	0.1883	0.081	0.01	0.02723	0.214438	0.045951	0.045951	0.0681
0.075	0.253	− 0.04893	0.0664	0.0597	0.255486	− 0.058933	0.10949	0.071341	0.1909	0.082	0.01	0.02739	0.215929	0.04627	0.04627	0.0685
0.125	0.256	− 0.04921	0.0689	0.06	0.261905	− 0.059437	0.112245	0.072292	0.196	0.084	0.01	0.02775	0.218925	0.046912	0.046912	0.0693
0.15	0.262	− 0.04934	0.07016	0.0601	0.26516	− 0.059709	0.11364	0.072761	0.1986	0.085	0.01	0.02794	0.220416	0.047232	0.047232	0.0697

Table 9 The effect of changing the parameter c_1 on the total profit of the supply chain

c_1	No alliance				RV alliance				RS alliance				RSV alliance			
	π_R	π_S	π_V	θ'	π_R	π_S	π_V	θ'	π_R	π_S	π_V	θ'	π_R	π_S	π_V	θ'
0.2	0.2896	− 0.03446	0.05442	0.03862	0.27952	− 0.021774	0.119796	0.066097	0.258	0.1097	0.000152	0.0232	0.267218	0.057261	0.057261	0.057113
0.3	0.2704	− 0.04021	0.05493	0.04914	0.26743	− 0.042387	0.114615	0.068944	0.222	0.0951	0.004876	0.02539	0.238196	0.051042	0.051042	0.06302
0.5	0.2427	− 0.06461	0.06594	0.07054	0.25339	− 0.072294	0.108594	0.074687	0.173	0.0739	0.01554	0.02976	0.204918	0.043911	0.043911	0.074835
0.6	0.2318	− 0.08557	0.05858	0.08126	0.25154	− 0.081705	0.107805	0.07756	0.158	0.0678	0.02078	0.03168	0.199395	0.042727	0.042727	0.080742

Table 10 The effect of changing the parameter f_1 on the total profit of the supply chain

f_1	No alliance				RV alliance				RS alliance				RSV alliance			
	π_R	π_S	π_V	θ'	π_R	π_S	π_V	θ'	π_R	π_S	π_V	θ'	π_R	π_S	π_V	θ'
0.055	0.254	− 0.03188	0.06683	0.1136	0.259	− 0.063554	0.11101	0.071553	0.19894	0.08526	0.0083	0.02715	0.220472	0.047244	0.047244	0.0675
0.0825	0.254	− 0.04222	0.06761	0.1183	0.260	− 0.059396	0.11125	0.071825	0.1946	0.0834	0.00961	0.02748	0.218092	0.046734	0.046734	0.068623
0.1375	0.258	− 0.05542	0.06794	0.1203	0.260	− 0.06054	0.11091	0.071722	0.19285	0.08265	0.01019	0.02761	0.217112	0.046524	0.046524	0.069071
0.165	0.261	− 0.06138	0.06801	0.1206	0.260	− 0.062398	0.11110	0.071606	0.1925	0.0825	0.0103	0.02764	0.21693	0.046485	0.046485	0.069154

Table 11 The effect of the simultaneous change of λ_2 and μ on green degree

RV	λ_2			
	0.01	0.015	0.025	0.03
μ				
0.05	0.0345	0.08909	0.1988	0.2542
0.075	0.023	0.05936	0.1324	0.1691
0.125	0.0138	0.0356	0.07931	0.1012
0.15	0.0115	0.02967	0.06607	0.08432
N	λ_2			
	0.01	0.015	0.025	0.03
μ				
0.05	0.06856	0.10289	0.17164	0.20609
0.075	0.042005	0.062995	0.10503	0.12607
0.125	0.023065	0.0346	0.057673	0.069214
0.15	0.018678	0.028018	0.04667	0.056044
RS	λ_2			
	0.01	0.015	0.025	0.03
μ				
0.05	0.02755	0.04136	0.0692	0.08327
0.075	0.01838	0.02759	0.04608	0.0554
0.125	0.01101	0.01654	0.0276	0.03315
0.15	0.009176	0.01378	0.02298	0.0276
RSV	λ_2			
	0.01	0.015	0.025	0.03
μ				
0.05	0.085355	0.19105	0.24434	0.03279
0.075	0.056867	0.12715	0.16249	0.021852
0.125	0.034103	0.076184	0.097296	0.013107
0.15	0.028416	0.063465	0.081039	0.010922

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Key Questions

1. What will be the best strategy and return policy during disruption in supply chain management ?
2. How can we obtain the flexibility in SCM by defining a good alliance strategy?
3. What will be the effect of green products and return policy in the final price for the customers?

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