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Fault detection in sensor information fusion Kalman filter

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Abstract

An approach to the test of the sensor information fusion Kalman filter is proposed. It is based on the introduced statistics of mathematical expectation of the spectral norm of a normalized innovation matrix. The approach allows for simultaneous test of the mathematical expectation and the variance of innovation sequence in real time and does not require a priori information on values of the change in its statistical characteristics under faults. Using this approach, fault detection algorithm for the sensor information fusion Kalman filter is developed.

Keywords: Fault detection; Kalman filters; Parallel algorithms; Sensor fusion; Matrix norm

1. Introduction

The necessity for operative test of the Kalman filter arises in many problems concerned with navigation and control [1,2]. Some algorithmic methods have been developed for this purpose. The algorithmic techniques for testing the Kalman filter are reviewed in [3] from which it follows that, by this way one can not only ensure failure localization and detection but also estimate correction. At present there are number of the algorithmic methods for the testing of Kalman filter [1–7], which is used for fault detection in Kalman filter different diagnostic signs. In spite of the great variety of the algorithmic methods for testing of Kalman filter, to date, questions of monitoring and diagnostics of its multi-channel modification (sensor information fusion Kalman filter) are not investigated.

In many applications it is possible to receive information on the state vector of a dynamic system from several sources simultaneously (as an integrated navigation system). Integrated navigation systems are still used in various applications successfully. In the aerospace and navy navigation systems, GPS, DGPS, GLONASS and INS systems are integrated in different combinations via Kalman filtering [5,8–15]. Federated [16–18] or parallel [19,20] Kalman filters are satisfactorily used to integrate different navigation systems. These kinds of filters are known as multi-channel or sensor information fusion Kalman filters. Algorithms have been developed for multi-channel estimation of the system parameters and state, which use for the estimation a mathematical model of a dynamic system, as well as measurements of several measurement channels (sensor information fusion Kalman filters). In these types of Kalman filters, simultaneous processing of the available data allows one to improve the estimation accuracy of the state vector and the reliability of data processing. Application of these algorithmic techniques for testing the sensor information fusion Kalman filters is concerned with a considerable increase of the required amount of computation because each estimation channel requires its own “failure detector”. Taking into account that the multi-channel estimation procedure also

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requires a great amount of computation, implementation of the test of the sensor information fusion Kalman filter by the previous technique is not an easy problem. Hence, it is necessary to develop simple algorithms for test of the multi-channel estimation procedures in order to perform real-time test of the filter without a priori information on changes in its parameters under fault. In this paper, the algorithmic testing of the sensor information fusion Kalman filter is investigated. This problem is solved by using the introduced statistic of mathematical expectation of spectral norm of the normalized innovation matrix of the mentioned Kalman filter. The approach allows simultaneous test of the mathematical expectation and the variance of the normalized innovation sequence.

2. Innovation approach to test of the sensor information fusion Kalman filter

Let us consider a linear dynamic system given by the equation of state

$$x(k+1) = \Phi(k+1,k)x(k) + G(k+1,k)w(k) \tag{1}$$

where $x(k)$ is the N-dimensional state vector of system, $\Phi(k+1,k)$ is the $N \times N$ transition matrix of the system, $w(k)$ is the random N-dimensional perturbation vector, $G(k+1,k)$ is the $N \times N$ transition matrix of perturbations (noise of the system). The process $x(k)$ is observed by a multi-channel system consisting of m measurement channels; the equation of measurements for the ith channel has the form

$$z_i(k) = H_i(k)x(k) + v_i(k) \tag{2}$$

where $z_i(k)$ is the n-dimensional measurement vector of measurements of the ith measurement channel, $H_i(k)$ is the $n \times N$ measurement matrix of the system for the ith channel. Measurement noises of the ith channel $v_i(k)$ have a zero vector of means and the correlation matrix $E[v_i(k)v_i^{\mathrm{T}}(j)] = R_{ii}(k)\delta(kj)$, they are uncorrelated in individual channels.

The state vector of the system can be estimated by using a sensor information fusion Kalman filter [5], which is characterized by

$$\hat{x}(k/k) = \hat{x}(k/k-1) + \sum_{i=1}^{m} P(k/k)H_i^{\mathrm{T}}(k)R_{ii}^{-1}(k)\Delta_i(k) \tag{3}$$

where $\hat{x}(k/k-1)$ is the extrapolation value, Δ_i is the innovation sequence of the ith channel

$$\Delta_i(k) = z_i(k) - H_i(k)\hat{x}(k/k-1) \tag{4}$$

The correlation matrix of the filtration error is given as

$$P^{-1}(k/k) = P^{-1}(k/k-1) + \sum_{i=1}^{m} H_i^{\mathrm{T}}(k)R_{11}^{-1}(k)H_i(k) \tag{5}$$

The correlation matrix of the extrapolation error is calculated as

$$\begin{aligned} P(k/k-1) &= \Phi(k,k-1)P(k-1/k-1)\Phi^{\mathrm{T}}(k,k-1) \\ &\quad + G(k,k-1)Q(k-1)G^{\mathrm{T}}(k,k-1) \end{aligned} \tag{6}$$

The sensor information fusion filter corresponding to Eqs. (3)–(6) is called a parallel filter since the optimal estimate in this case is obtained by simultaneous weight summation of the innovation sequences $\Delta_i(k)$ of all channels with the use of the common extrapolator. As is seen from (3)–(6), implementation of the parallel technique for estimating the state vector requires m filters, and for each filter the gain matrix, correlation matrix of filtering errors, etc. must be calculated. For normal functioning of the sensor information fusion Kalman filter (3)–(6), the normalized innovation sequences of different channels

$$\tilde{\Delta}_i(k) = [H_i(k)P(k/k-1)H_i^{\mathrm{T}}(k) + R_{ii}(k)]^{-1/2}\Delta_i(k) \tag{7}$$

obey the distribution $N(0,1)$ [21]. Faults leading to an abrupt change in the measurement channel characteristic, computer failures, anomalous measurements, changes in the statistical characteristics of measurement or object noise, and a variance of trajectories of the real process and the estimates generated by the Kalman filter, etc. result in changes in the previous characteristics of the sequence $\tilde{\Delta}(k)$. It is of interest to develop an operative method for simultaneous check of the mathematical expectation and the variance of the normalized innovation sequence (7). For this purpose, two hypotheses are set up: γ_0 – Kalman filter operates normally and γ_1 – a fault occurs in the estimation system. Let us introduce the following definition.

Definition. A rectangular $n \times m$ matrix (n is the innovation vector dimension; $n \geqslant 2$; $m \geqslant 2$) is defined as a normalized innovation matrix of the m-channel Kalman filter, in which columns are represented by normalized innovation vectors of different channels, corresponding to the same moment of time (dimensions of the normalized innovation vectors of different channels are assumed to be equal), i.e.,

$$A(k) = [\tilde{\Delta}_1(k), \tilde{\Delta}_2(k), \ldots, \tilde{\Delta}_m(k)] \tag{8}$$

To test the hypotheses γ_0 and γ_1, the matrix $A^{\mathrm{T}}(k)A(k)$, which obey the Wishart distribution, can be used. There exists some interesting results on the eigenvalues and maximum and minimum eigenvalues of Wishart distributed matrices [22–24]. But application of mentioned works to fault detection problem of multidimensional dynamic systems turns out to be very complicated since there are difficulties in determining the confidence domain (or intervals) for the eigenvalues of random matrix.

In this study to test the hypotheses γ_0 and γ_1, the spectral norm of the matrix $A(k)$ is used. As is known [25], the spectral norm $\| \|_2$ of the real matrix $A(k)$ is defined by the formula:

$$\|A(k)\|_2 \equiv \max\{(\lambda_i[A^{\mathrm{T}}(k)A(k)])^{1/2}\} \tag{9}$$

where $\lambda_i[A^{\mathrm{T}}(k)A(k)]$ are eigenvalues of the matrix $A^{\mathrm{T}}(k)A(k)$.

Square roots of eigenvalues of the matrix $A^{\mathrm{T}}(k)A(k)$, i.e., the values $\lambda_i[A^{\mathrm{T}}(k)A(k)]$, are called singular values of the matrix $A(k)$. Therefore, the spectral norm of the matrix $A(k)$ is equal to its maximal singular value. The singular numbers are real and nonnegative [25]. Due to the same reasons, determination of the singular numbers and, consequently, the spectral norm is a simpler problem in terms of computing than obtaining eigenvalues for a arbitrary matrix. This explains the choice of the spectral norm of the normalized innovation matrix of Kalman filter as a scalar measure to be tested. To test the hypotheses γ_0 and γ_1, a one-dimensional statistic of mathematical expectation of the spectral norm of the matrix $A(k)$ is introduced. As an estimate of mathematical expectation its arithmetic mean will be used. For a great value of k the following expression may be written as

$$E\{\|A(k)\|_2\} \approx \overline{\|A(k)\|_2} = \frac{1}{k}\sum_{j=1}^{k} \|A(j)\|_2 \tag{10}$$

Hansen [26] has found a number of bounds for mathematical expectation of the spectral norm of the random matrix $A(k) \in R^{n\times m}$ composed of normally distributed random values with zero mathematical expectation and the standard deviation $\sigma(a_{ij} \in N(0,\sigma))$. Let us consider some of them. Assume that r_i^{T} and a_j denote rows and columns of matrix A. Let us introduce the maximal row-column norm

$$\mu \equiv \max[\|r_i\|_2, \|a_j\|_2] \tag{11}$$

where $\|r_i\|_2$ and $\|a_j|_2$ are the corresponding Euclidean vector norms. The following inequality is true [26]:

$$E\{\mu\} \leqslant E\{\|A\|_2\} \leqslant [\max(n,m)]^{1/2} E\{\mu\} \tag{12}$$

It is not easy to use formula (12) in practical calculation because it is difficult to estimate $E\{\mu\}$. That is why $E\{\mu\}$ is replaced by its lower bound

$$\sigma\sqrt{\max(n,m)} = \max[E\{\|r_i\|_2\}, E\{\|a_j\|_2\}] \leqslant E\{\mu\} \tag{13}$$

Eq. (12) will be written in the following form:

$$\sigma\sqrt{\max(n,m)} \leqslant E\{\|A\|_2\} \leqslant f(\max(n,m)) \times \sigma\sqrt{\max(n,m)} \tag{14}$$

where f is the unknown function to be defined. By mathematical computer simulation it was shown in [26] that the value $\sigma\sqrt{\max(n,m)}$ is an adequate lower bound for $E\{\|A(k)\|_2\}$. By numerical calculation it was also shown that the function f at $n=m\to\infty$ approaches 2 asymptotically and f is always between the values 1 and 2. Therefore the value 2 is proposed for an estimate of the function f. In view of the foregoing, the following simple bounds for $E\{\|A(k)\|_2\}$ can be obtained:

$$\sigma\sqrt{\max(n,m)} \leqslant E\{\|A\|_2\} \leqslant 2\sigma\sqrt{\max(n,m)} \tag{15}$$

Expression (15) characterizes the relation between the standard deviation σ of elements of the random matrix A and its spectral norm. Taking into account that the normalized innovation matrix $A(k)$ used in order to detect sensor information fusion Kalman filter faults consists of normally distributed random elements with zero mathematical expectation and the finite variance $a_{ij} \in N(0,1)$, inequality (15) can be applied to solve the diagnostic problem formulated in this paper. It can be stated that if the elements a_{ij} of the normalized innovation matrix of the Kalman filter obey the distribution $N(0,1)$, then inequality (15) must be fulfilled. No fulfillment of inequality (15) is an evidence that the zero mean of the elements a_{ij} is biased, or the unit variance has changed, or $\{a_{ij}\}$ differs from white noise.

The sensor information fusion Kalman filter will be tested upon fulfillment of inequality (15) that in view of $\sigma=1$ and expression (10) will be written in a simpler form

$$\sqrt{\max(n,m)} \leqslant \overline{\|A(k)\|_2} \leqslant 2\sqrt{\max(n,m)} \tag{16}$$

Hence, while solving the problem of testing the sensor information fusion Kalman filter (3)–(6), the decision rule with respect to the hypotheses must have the form

$$\begin{aligned}
&\gamma_0 : \sqrt{\max(n,m)} < \overline{\|A(k)\|_2} < 2\sqrt{\max(n,m)}, \quad \forall k,\\
&\gamma_1 : \text{if } \exists k, \text{ where } \overline{\|A(k)\|_2} \leqslant \sqrt{\max(n,m)},\\
&\quad \text{or } \overline{\|A(k)\|_2} \geqslant 2\sqrt{\max(n,m)}
\end{aligned} \tag{17}$$

The determined bounds for mathematical expectation of the spectral norm of the normalized innovation matrix of the m-channel Kalman filter are sufficiently simple and allow

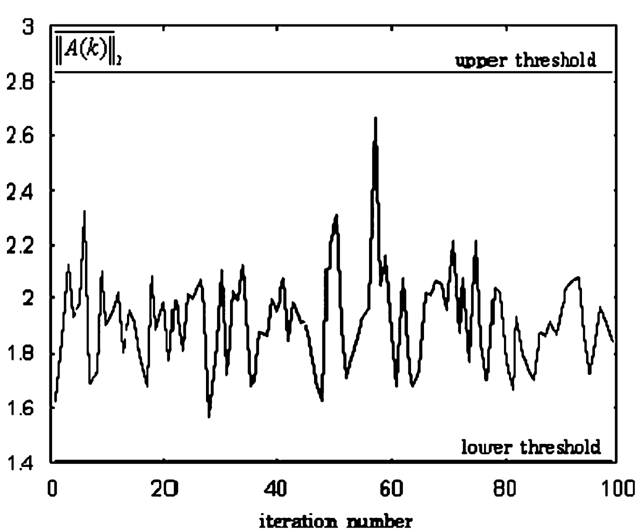

Fig. 1. Graph of the statistic $\overline{\|A(k)\|_2}$ for normal operating of the measurement channels.

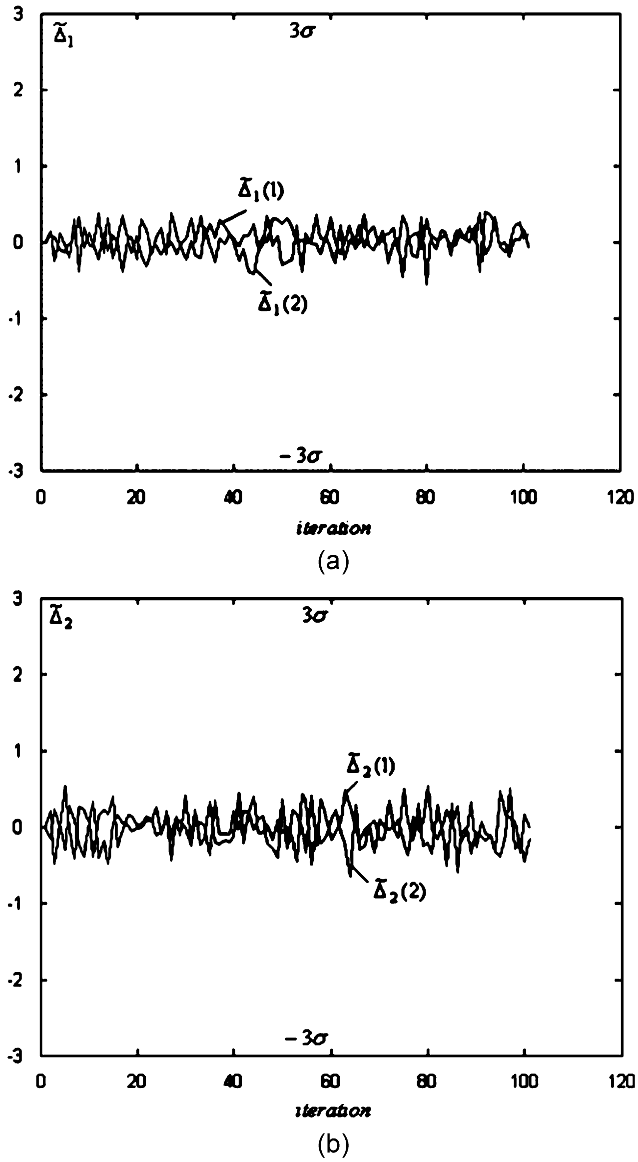

Fig. 2. Behaviors of the normalized innovation sequences in the case of normal operating of the measurement channels: (a) for the first estimation channel and (b) for the second estimation channel.

for simultaneous check of mathematical expectations and variances of the innovation sequences of all the channels.

In real exploitation conditions of the system, the test of the sensor information fusion Kalman filter algorithm is reduced to the following sequence of calculations performed at each measurement step.

1. Using (3)–(6) and (7) calculate the multichannel Kalman estimate of the state vector of the system and the values of vectors of the normalized innovation sequences of different channels at the step k.
2. Compose the normalized innovation matrix of the m-channel parallel sensor information fusion Kalman filter for given $n \geqslant 2$ and $m \geqslant 2$ in the form (8).

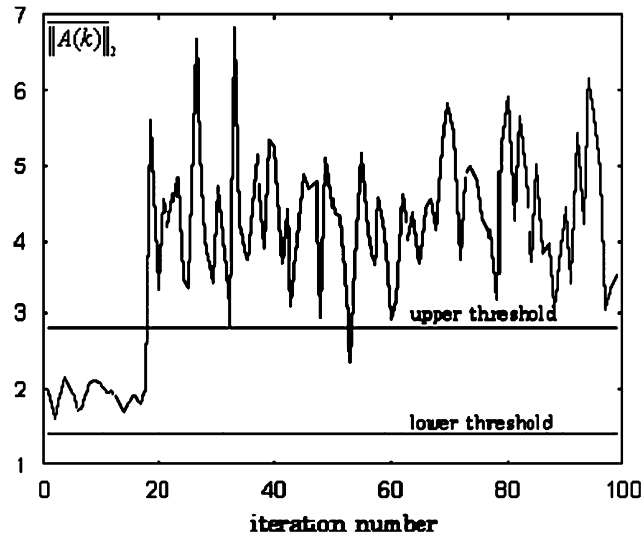

Fig. 3. Graph of the statistic $\overline{\|A(k)\|_2}$ in the case of bias in the second measurement channel.

3. Determine the eigenvalues of the matrix $A^{\mathrm{T}}(k)A(k)$ and the spectral norm of the matrix $A(k)$.
4. Calculate the statistic of arithmetic mean of the spectral norm $\|A(k)\|_2$ using formula (10).
5. Check fulfillment of inequality (16) and make a decision on normal functioning of the sensor information fusion Kalman filter from the decision rule (17).
6. Repeat the sequence of calculations beginning from operation 1 for the next moment of time $k+1$.

3. Simulation results

Let us consider the two-dimensional two-channel dynamic system given by equation of state (1) and measurements for the ith channel:

$$z_i(k) = H_i(k)x(k) + v_i(k), \quad i = 1, 2 \tag{18}$$

The system parameters are written in the following form:

$$\phi(k+1,k) = \begin{bmatrix} 0,5 & 0,816 \\ -0,6 & 0,4 \end{bmatrix}, \quad H_i(k) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$i = 1, 2;$$

$$Q(k) = \begin{bmatrix} 0,1 & 0 \\ 0 & 0,1 \end{bmatrix}, \quad R_{11} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$R_{22} = \begin{bmatrix} 0.64 & 0 \\ 0 & 0.64 \end{bmatrix}$$

Measurements were processed using the sensor information fusion Kalman filter (3)–(6) (two-channel, in this case) by parallel implementation, i.e., the optimal estimate of the state vector has been obtained by simultaneous weight summation of the innovation sequences $v_i(k)$, $i = 1, 2$, of two channels with the use of the common extrapolator. According to definition above the normalized innovation 2×2 matrix of the

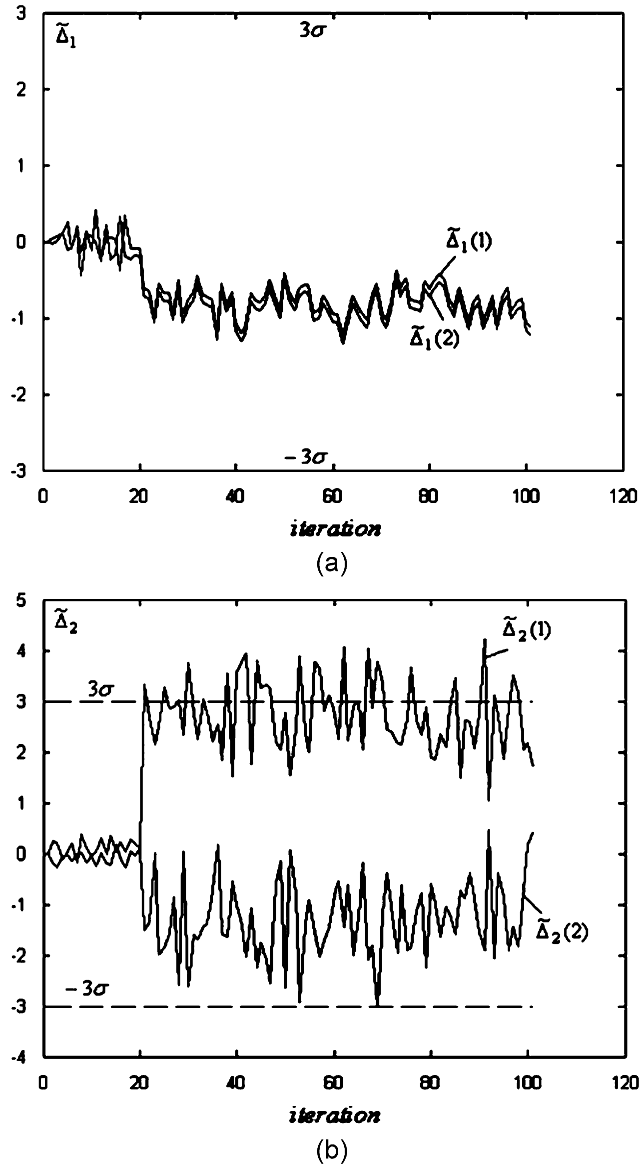

Fig. 4. Behavior of the normalized innovation sequences in the case of bias in the second measurement channel: (a) for unimpaired estimation channel and (b) for impaired estimation channel.

two-channel Kalman filter is formed: $A(k)=[\tilde{\Delta}_1(k), \tilde{\Delta}_2(k)]$. The spectral norm of the innovation matrix and its mathematical expectation were calculated by means of (9) and (10). The decision rule for the case under consideration $n=2$, $m=2$ (m is the number of channels) is written in the form

$$\gamma_0 : \sqrt{2} < \overline{\|A(k)\|_2} < 2\sqrt{2}, \quad \text{for } k=2,3,\ldots$$
$$\gamma_1 : \text{if } \exists k, \text{ where } \overline{\|A(k)\|_2} \leqslant \sqrt{2}, \text{ or } \overline{\|A(k)\|_2} \geqslant 2\sqrt{2} \quad (19)$$

Obtained results are presented in Figs. 1–6. Fig. 1 shows admissible bounds of the statistic $\overline{\|A(k)\|_2}$ and a plot of its behavior in the case of normal functioning of the both filtering channels. The corresponding normalized innovation sequences are shown in Fig. 2(a) and (b). To verify efficiency

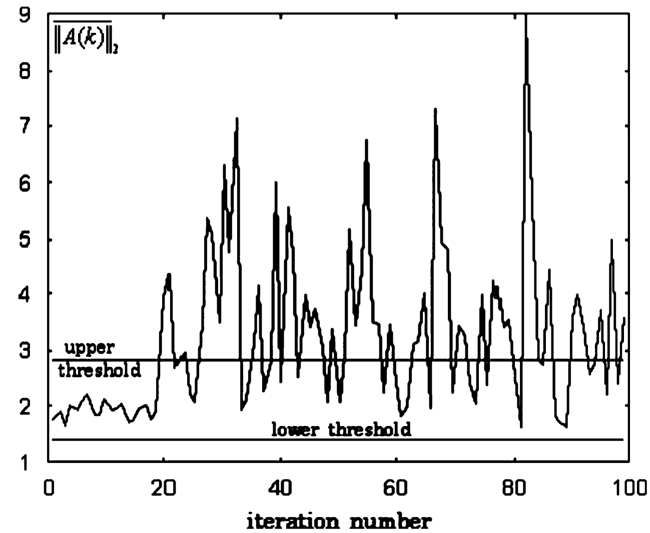

Fig. 5. Graph of the statistic $\overline{\|A(k)\|_2}$ in the case of change in the measurement noise variance of the second measurement channel.

of proposed algorithm, beginning from the step $k=20$, a fault in the first measurement sensor of the second measurement channel is simulated, introducing a constant bias as follows:

$$z_{21}(k)=x_1(k)+3+v_{21}(k) \quad (k\geqslant 20) \quad (20)$$

As it is seen from expression (20) the constant bias was entered by adding 3 to the measurements noises $v_{21}(k)$ in the first measurement sensor of the second measurement channel. The simulation results corresponding to this case are presented in Figs. 3 and 4. The corresponding results presented in Fig. 3 shows that the value of the statistic $\overline{\|A(k)\|_2}$, beginning from the 20th step, grows abruptly, and at the step $k=22$ it exceeds the upper admissible bound. This fact evidences fault detection in the two-channel sensor information fusion Kalman filter. Behavior of the normalized innovation sequences in the case of bias in the second measurement channel is presented in Fig. 4(a) and (b).

The performed simulations show that the detected minimum fault rate is 4% of the measurement value when the mean of the normalized innovation sequence is tested and the detection time for this is 4 s.

To acknowledge the ability of checking normalized innovation sequence variance by means of the developed algorithm, beginning from the step $k=20$ the values of the measurements noises $v_{21}(k)$ of the first measurement sensor of the second measurement channel were changed by multiplying them by 3 as follows:

$$z_{21}(k)=x_1(k)+3v_{21}(k) \quad (k\geqslant 20) \quad (21)$$

The obtained results are given in Figs. 5 and 6(a) and (b). Plots in Fig. 5 show that the value of the statistic $\overline{\|A(k)\|_2}$ in this case after the 20th step grows abruptly, and at the step $k=23$ it exceeds its upper admissible bound. As a result, using the decision rule (19) the fault in the two-channel

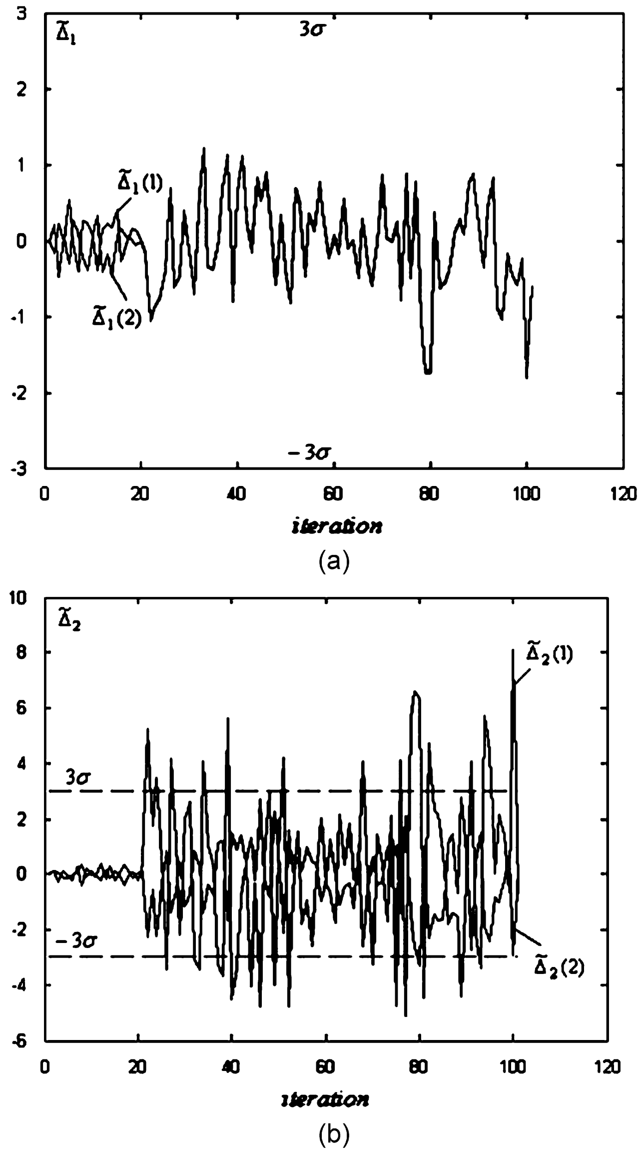

Fig. 6. Behavior of the normalized innovation sequences in the case of change in the measurement noise variance of the second measurement channel: (a) for unimpaired estimation channel and (b) for impaired estimation channel.

sensor information fusion Kalman filter is detected. The behavior of the appropriate normalized innovation sequences is presented in the Fig. 6(a) and (b). The performed simulations in this case show that the detected minimum fault rate is 30% of the standard deviation of measurement noises when the variance of the normalized innovation sequence is tested and the detection time for this fault rate is 1.6 s.

4. Conclusion

Using the statistic of mathematical expectation of the spectral norm of the normalized innovation matrix the approach to sensor information fusion Kalman filter test is proposed. The approach allows real-time simultaneous check of mathematical expectation and variance of the innovation sequence and does not require a priori data on the values of change in its statistical characteristics in the case of fault. The upper and lower bounds of the statistic have been found. They are determined by dimension of the measurement vector and by the number of the filtering channels. The algorithm of sensor information fusion Kalman filter test is adaptable to the change in the number of estimation channels and allows to detect faults in real time.

The proposed algorithm can be used when the dimensions of the innovation vectors of different channels are not equal. In this case the normalized innovation vectors, which have dimensions less than n (n is the maximum dimension of the used innovation vectors), should be completed with the $N(0, 1)$ distributed random numbers when composing the $(n \times n)$ normalized innovation matrix of the m-channel parallel sensor information fusion Kalman filter.

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