ISSN 2422-8702 (Online), **DOI:** 10.7176/JSTR/5-9-02
Vol.5, No.9, 2019

Finding Optimal Number of Ship with the Greatest Integer Function in Maritime Transportation

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Abstract
Various parameters are taken into account when calculating fleet in maritime transport. The most important of these is fuel consumption. Since the fuel consumption is proportional to the speed of the ship, high speed brings high fuel consumption. Increasing number of ship means increasing costs. In this study, we use the greatest integer function to find the optimum number of ship with lower cost. In addition, we calculate daily freight for voyage charterer depending on the number of ships by using this function.

Keywords: Maritime Transportation, Number of Fleet, Fuel Consumption.

DOI: 10.7176/JSTR/5-9-02

1. Introduction

Experimental data show that fuel consumption varies geometrically with increasing speed. For example, at some speeds, when you increase your ship's speed by 30%, your consumption is twice as fast as the original speed. While the vessels are anchored in the port, roughly a quarter of the fuel consumption in the sea is produced by fuel fuels [1]. In the case of neglecting the load on it, there is a classical correlation which is proportional to the power to increase the speed and fuel consumption by showing the fuel consumption of a ship with F. That is,

$F(v) = \lambda . v^{\Omega}$

$\lambda > 0$ [2]

However, since there are some fixed costs to show with s, a daily cost of a ship is $C_1 = s + F(v)$. Similarly, the 1-day cost of n homogenous ships is $C_n = n(s + F(v)) = n.\ C_1$ [3]

As can be seen, the number of ships and the cost are increasing in direct proportion, but if the speed is inversely proportional to the strength of the speed, the cost can be further reduced by increasing the number of ships by only 1 and reducing the speed to a very small amount.

Example 1.1

For a ship with fixed cost s = \$ 1000 and speed v = 20,1 knots, $\lambda = 0{,}64$ means that a 1(one) day cost is $C_1 =$ \$ 6197. For two smaller vessels with the same characteristics but with a speed v = 20 knots, the cost would be $C_2 =$ \$ 12240. At first glance it may seem more costly to ship 2 slower and smaller vessels, but the increase in the number of vessels will cause the cost to decrease. For example, for n = 10 vessels, C_{10} = \$ 61970, $C_{11} =$ \$ 67320. For the convenience of calculation, $\Omega = 3$ is taken for convenience, and the fixed cost is considered unaffected by a very small reduction of 0.1 knots. In Table 1, ship number relationships are given for s = \$ 1000 and $\lambda = 0.64$. [4]

Table1: Regretion with s = $ 1000 and λ= 0,64 [4]

n:	**C_n at 20,1 kt**	**C_{n+1} at 20 kt**
1	$6197,18	$12240
2	$12394,36	$18360
3	$18591,55	$24480
10	$61971,84	$67320
50	$309859,23	$312120
80	***$495774,77***	***$489600***
81	***$501971,96***	***$495720***

As shown in the table above, the cost of 80 ships at 20.1 kt is greater than the speed of 81 ships at 20 kt. In other words, although we have increased the number of vessels, our costs have been reduced by only 0.1 kt slowdown.

2. The Greatest Integer Function

For all real numbers, *x*, the greatest integer function returns the largest integer less than or equal to *x*. In essence, it rounds down a real number to the nearest integer. It is denoted by [.]

$f:\mathbb{R}\longrightarrow\mathbb{Z}$, $f(x)=[x]$.

The greatest integer function has these properties:

(i) $k \in \mathbb{Z}$ such that $f(x) = k \Leftrightarrow k \leq x < k+1$

(ii) For all $x \in \mathbb{R}$ and $k \in \mathbb{Z}$ $f(x+k) = f(x) + k$

(iii) For all $x \in \mathbb{R}$ i$f(x) \leq x < f(x) + 1$

Proposition 2.1

C'_1 and C_1 are the costs of two ships which are slow and fast respectively. Let κ be the greatest integer function and let n be the number of the ship. The smallest natural number n providing the inequality of $(n+1).\ C'_1 < n.C_1$ is $\kappa\left(\frac{C'_1}{C_1-C'_1}\right)+1$ This equation does not depend on the changing of the fixed cost ***s***. [4]

Proof:

We know that $C'_1 = s' + F(v')$ $C_1 = s + F(v)$

Let $(n+1).C'_1 < n.\ C_1$ be obtained.

$n.C'_1 + C'_1 < n.C_1$

$C'_1 < n.(C_1 - C'_1)$

$$\frac{C'_1}{C_1 - C'_1} < n$$

According to the third property of the greatest integer function, the smallest natural number n providing the inequality is $n = \kappa\left(\frac{C'_1}{C_1-C'_1}\right)+1$

If we use this proposition, we can calculate the number of the ship in example 1.

$n = \kappa\left(\frac{C'_1}{C_1-C'_1}\right)+1=\left[\frac{6120}{77}\right]+1 = [79{,}48]+1 = 80$ so, C_{80} = $495774,77 and C'_{81} = $495720. We showed that cost of 80 ships are greater than 81 ships in the table 1 before. So, we can calculate this by using Proposition 2.1.

In some cases it may be less costly to do a few times with smaller vessels. To generalize this situation, the optimal number of vessels can be found by the following proposition.

Proposition 2.2

Let C_1 and C'_1 functions be the consumption functions of the two large and small ships respectly. Let κ be the greatest integer function. For m>n , the smallest natural number n providing the inequality $n.C_1 > m.C'_1$ is $\kappa\left(\frac{C'_1}{C_1-C'_1}\right)+1$.

Proof:

Let $n.C_1 > m.C'_1$ be obtained. for m>n . and s' be the fixed cost of C'_1 . So,

$m.(s'+F(v')) < n.s + n.F(v)$

$\Rightarrow m.s' + m.F(v') < n.s + n.F(v)$

m>n obtains m=n+k, $k \in \mathbb{Z}+$

$n.s' + n.F(v') + k.s' + k.F(v') < n.s + n.F(v)$

$\Rightarrow k.s' + k.F(v') < n.s + n.F(v) - n.s' - n.F(v')$

$\Rightarrow$ k.(s' + F(v ')) < n.(s + F(v) -(s' + F(v ')))

$\Rightarrow \frac{n}{k} > \frac{C'_1}{C_1 - C'_1}$

$\Rightarrow \frac{n}{m-n} > \frac{C'_1}{C_1 - C'_1}$.

the smallest natural number n providing the equality is $n = \kappa\left(\frac{C'_1}{C_1 - C'_1}\right) + 1$

3. Voyage Charter

Instead of increasing the number of ships, it will be more profitable to carry the maximum load in the shortest time by reducing the time of the voyage for voyage charter. In other word, we must increase the speed.

$16\lambda Pv^3 + \lambda DV^2 = 8F$ equation is valid for this transportation. Here D, Number of days at sea; P stands for the number of days in port and F stands for freight earned. In particular, if we take $P = 0$, we get the equation $kDV^2 = 8F$ [1]

Example 3.1

Let λ be the constant for the ship machine and v be the speed of the ship. If λ = 0,5 and v = 10 kt. then earned freight is 8F = \$50. But, if we increase the speed by 0.5 knots, earned freight will be 8F = \$55; 125. [1]

Proposition 3.2

Let κ be the greatest integer function $v_1$and v_2 are the speeds of the ship, D be the number of the day. and $v_1 < v_2$ If we take $D = \kappa(\frac{v_2{}^2}{v_2{}^2 - v_1{}^2})$ then $(D-1)v_2{}^2 < D.v_1{}^2$ inequality is obtained.

Proof:

If we consider $(D-1)v_2{}^2 < D.v_1{}^2$ inequality, we can divide by D and we get $D < \frac{v_2{}^2}{v_2{}^2 - v_1{}^2}$ The smallest D number that satisfies this condition is $D = \kappa(\frac{v_2{}^2}{v_2{}^2 - v_1{}^2})$.

Now we can use this proposition on example 2.

If we take $D = \kappa\left(\frac{v_2{}^2}{v_2{}^2 - v_1{}^2}\right) = \kappa\left(\frac{110{,}25}{10{,}25}\right) = 10$, we have $10 \times 0{,}5 \times 10^2 = 500 = 8F$

And for $(D-1)$ day, $9 \times 0{,}5 \times 10{,}5^2 = 496{,}125 = 8F$ that means spending longer days with lower speed is more profitable than spending shorter days with higher speed

4. Literature Review.

Carlou (2011) calculated effects of relation of CO2 emisison and reducing of speed which included various factors such as size, speed number of days at sea, seaport, fuel consumption for container vessels [5]. Christiansen et al (2012) compiled to 131 articles which are published journals about subject of ship routing and scheduling between 2002-2012. They presented methods which used in their study [6]. Kim et al (2012) determined amount of fuel and optimum vessel speed for a specific vessel route. The study was solved the problem by using epsilon-optimal algorithm [7]. Notteboom and Cariou (2013) researched effects of slow speed applications. Also they analyzed fuel consumption and BAF which paid by shippers [8]. Khor et al (2013) set up a software to optimize speed of ultra-container vessels. They found out optimum speed as 19.5 knot [9]. Sheng et al (2014). Fagerholt et al (2010) solved the optimal speed on a specific route problem [10]. Mersin and Nur (2019) built a model by using the greatest integer function.

5. Results

In the first part of the study a very small decrease in speed has been shown to have a positive effect on increasing the number of ships. We can calculate optimum number of the ship with “greatest integer function” which is represented by κ in this study. It has been shown that the smallest number of vessels providing $n.C_1 > m.C'_1$inequality for n> m, which is the consumption functions of two large and small vessels, respectively, $C_1$and C'_1, is $\kappa\left(\frac{C'_1}{C_1 - C'_1}\right) + 1$ In this way, 20 kt. 81 ships departing at a cost of \$ 495774.77 from 20.1 kt. The cost of 80 vessels traveling quickly was found to be \$ 495720.

In the second part, it is seen that the same function can be applied for a voyage charter.

In addition, we see that spending longer days with lower speed is more profitable than spending shorter days with higher speed for voyage charter.

References

[1] Alderton, P. M. (1981). The optimum speed of ship. The Journal of Navigation, 34(3), 341–355.

[2] Wang,S., Meng, Q., 2012. Sailing speed optimization for container ships in a liner shipping network, Transportation Research Part E, Vol 48 pp.701-714

[3] Chrzanowski, I., 1989, An Introduction to Shipping Economics, Fairplay PublicationNL.T.D, United Kingdom.

[4] Mersin K.,2016, A Modelling of Optimal Speed and Route In Maritime Transport, PhD. Thesis, Istanbul University, Institute of Graduate Studies in Science and Engineering

[5] CarIou, P. (2011). Is slow steaming a sustainable means of reducing CO emissions from container shipping?, Transportation Research Part D, Vol.16, pp. 260-264.

[6] Christiansen, M., Fagerholt, K., Nygreen, B., Ronen, D., (2013). Ship routing and scheduling in the new millennium, European Journal of Operational Research, Vol 228, 3, pp.467–483.

[7] Kim, H., Chang., Y., Kim, K., Kim, H., (2012). An epsilon-optimal algorithm considering greenhouse gas emissions for the management of a ship's bunker fuel, Transportation Research Part D, Vol.17, pp.97-103.

[8] Notteboom, T., Cariou, P. (2013). Slow steaming in container liner shipping: is there any impact on fuel surcharge practices?, The International Journal of Logistics Management, Vol 24, 1, pp.73-86.

[9] Khor, Y.S., Dohlie, K. A., Konovessis, D., Xiao, Q. (2013). Optimum Speed Analysis for Large Containerships, Journal of Ship Production and Design, Vol 29, 3, pp.93-104.

[10] Fagerholt, K., Laporte, G. and Norstad, I. (2010). Reducing fuel emissions by optimizing speed on shipping routes, Journal of the Operational Research Society, Vol 6, 1, pp.523–529.

[11] Mersin, K., Nur,C. (2019). Mathematical Model of The Traveling Salesman Problem With Delivery And Draft Limit, Internatıonal Journal of Engıneerıng Technologıes-IJET, Vol 5, 3, pp. 78-83.